THE EXPERIMENT
INTERNATIONAL JOURNAL OF SCIENCE AND TECHNOLOGY

ISSN 2319-2119

RESEARCH ARTICLE

Amro Shalaan, The Experiment, 2016, Vol.38(4), 2310-2312

On a new Trigonometric Formula

Abstract

In this short paper we given a new approximate definition of $\sin^{-1} x$ as afunction of $x, \sqrt{1-x^2}$ which avoids imaginary units and natural logarithm that found in Euler formula result for all $\sin^{-1} x \in \left[0, \frac{\pi}{2}\right]$, therefore it will be interest add to many applied fields .

Key words

New trigonometric formula, Euler formula, Maple , Trigonometric quantities .

1. Introduction

Trigonometry has very important applications in our life[1] , by this way we see that our formula is an important add to Math , this formula is an approximate equation deduced by studying the relationship between some trigonometric quantities in the interval $\left[0, \frac{\pi}{2}\right]$, to be the way to define $\sin^{-1} x$ as a function of ($x, \sqrt{1-x^2}$) without imaginary units or natural logarithm that found in the equation :

$$\sin^{-1} x = -iln(\sqrt{1-x^2}+ix). \quad (1)$$

which deduced by Euler formula[2] , our formula states that :

$$\sin^{-1} x \approx x + \left(1-\sqrt{1-x^2}\right)^{1.78}(1-0.43x) \quad , \quad \sin^{-1} x \in \left[0, \frac{\pi}{2}\right]. \quad (2)$$

with error $< 1 \times 10^{-3}$ in radians .

2. Material and methods

The formula of this paper can be deduced by studying some trigonometric quantities when $x \in \left[0, \frac{\pi}{2}\right]$,we also used x in degrees in this section .

Assume that $g(x) = x - \sin x - (1-\cos x)^2$. (3)

Maple shows that $g(x)$ has three important information when $x \in [0,90]$

a) $g(x) = 0$ when $x = 40.5043$

b) $g(x) > 0$ when $x < 40.5043$

c) $g(x) < 0$ when $x > 40.5043$

Equation (3) leads to $\lim_{x\to 0} g(x) = 0$ when $\lim_{x\to 0}(1- \cos x)^{\gamma} = 0$

When (γ) is apositive value , then $g(x) \propto (1-\cos x)^{\gamma}$ (4)

Information (a,b,c) leads to :

$g(x) = 0$ when $x` = 45$

$g(x) > 0$ when $x` < 45$

$g(x) < 0$ when $x` > 45$

When $x` = x + 4.4957$

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But we know that :

$\cos x` - sinx` = 0$ when $x` = 45$

$\cos x` - \sin x` > 0$ when $x` < 45$

$\cos x` - \sin x` < 0$ when $x` > 45$

In comparison we get :

$g(x) = 0$ when $\cos x` - sinx` = 0$

$g(x) > 0$ when $\cos x` - \sin x` > 0$

$g(x) < 0$ when $\cos x` - \sin x` < 0$

Then $g(x) \propto (\cos x` - \sin x`)$ (5)

From the facts (4),(5) :

$$g(x) \propto (\cos x` - \sin x`)(1 - \cos x)^{\gamma} \quad (6)$$

$$\therefore g(x) = k(\cos x` - \sin x`)(1 - \cos x)^{\gamma} \quad (7)$$

By simplifying with trigonometric addition identities when $x` = x + 4.4957$

$$\therefore g(x) = (\alpha \cos x - \beta \sin x)(1 - \cos x)^{\gamma} \text{ , } (\alpha, \beta) \text{ are constants} \quad (8)$$

Now from equations (3,8) :

$$x = \sin x + (1 - \cos x)^2 + (\alpha \cos x - \beta \sin x)(1 - \cos x)^{\gamma} \quad ; \quad x \in [0,90] \quad (9)$$

let $x = 90$ then $\beta \approx 2 - \frac{\pi}{2} \approx 0.43$

$$\because \alpha \cos 40.5043 = \beta \sin 40.5043 \quad \therefore \alpha \approx 0.37 \quad (10)$$

To move away from the values which near to $(x = 0, x = 90)$ to get γ, we find that $\gamma \in (1.4554\,, 1.4697)$ when $x \in (10\,, 80)$ respectively .

from equation (9) :

let $\emptyset_x = 1 - \cos x$

$$\therefore x = \sin x + \emptyset_x^2 + (0.37(1 - \emptyset_x) - 0.43 \sin x)\emptyset_x^{\gamma} \quad (11)$$

$$\therefore x = \sin x + \emptyset_x^2 + 0.37\emptyset_x^{\gamma} - 0.37\emptyset_x^{\gamma+1} - 0.43\emptyset_x^{\gamma} \sin x \quad (12)$$

After we known the values of γ:

$$\emptyset_x^{\gamma} \in \left[\emptyset_{10}^{\gamma_{10}}, \emptyset_{80}^{\gamma_{80}}\right] = [2.25 \times 10^{-3}, 0.76] \quad (13)$$

$$\emptyset_x^{\gamma+1} \in \left[\emptyset_{10}^{\gamma_{10}+1}, \emptyset_{80}^{\gamma_{80}+1}\right] = [3.43 \times 10^{-3}, 0.62] \quad (14)$$

$$\emptyset_x^2 \in [\emptyset_{10}^2, \emptyset_{80}^2] = [2.3 \times 10^{-4}, 0.68] \quad (15)$$

that means ($\emptyset_x^{\gamma}, \emptyset_x^{\gamma+1}, \emptyset_x^2$)gives values partly near to each other , then we can substitute for

($\emptyset_x^{\gamma}, \emptyset_x^{\gamma+1}, \emptyset_x^2$) with $\emptyset_x^c$ which approximately mid-point between them, from equation (12):

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$$x \approx \sin x + \emptyset_x^c + 0.37\emptyset_x^c - 0.37\emptyset_x^c - 0.43\emptyset_x^c \sin x \qquad (16)$$

$$\therefore x \approx \sin x + \emptyset_x^c(1 - 0.43\sin x) = \sin x + (1 - \cos x)^c(1 - 0.43\sin x) \qquad (17)$$

Now we can get $c \approx 1.78$

Finally when x in radians :

$$\therefore x \approx \sin x + (1 - \cos x)^{1.78}(1 - 0.43\sin x) , x \in \left[0.\frac{\pi}{2}\right] \qquad (18)$$

$\therefore \sin^{-1} y \approx y + \left(1 - \sqrt{1 - y^2}\right)^{1.78}(1 - 0.43y)$, $y \in [0,1]$, which includes the required.

References:

1.Glen Van Brummden , The mathematics of the heavens and the earth , the early history of trigonometry (Princeton university press ,2009) .

2.Alan Jeffrey , Hui-Hui Dai , Handbook of mathematical formulas and integrals 4th edition (Elsevier , 2008) .

Author &Affilation

Amro Samy Shalaan Al Adham

High institute of Engineering and Technology, Damietta , Egypt