

INVESTIGATION OF CARBON NANOTUBES INFLUENCE IN STRUCTURAL DYNAMIC BEHAVIOR OF CARBON EPOXY LAMINATES

ABSTRACT

Carbon epoxy reinforced with carbon fibers was used in several ways and became replaced with different types of materials. Change in the weight ratio and orientations of carbon fibers and CNTs are influenced the natural frequency of the main structure. Knowledge of composite behavior in structures gives us the ability to choose the best composite for specific part.

The laminate composite materials which reinforced with carbon fibers are used. CNTs replaced with fibers in different volume ratios, and the stiffness matrixes are calculated. The carbon nano tubes are supposed to be long fibers and carbon fibers and CNTs are parallel. The influence of changing volume percentage of CNTs and orientation of reinforcements are numerically analysis. Specific cantilever laminate beams with orthogonal layers are modeled and analysis with analytical method and Abaqus software. Specific Abaqus results are checked with analytical solution for bending vibrations and the Abaqus results are validated.

The results obtained show that, how change in orientation of compounds change the natural frequency and vibration behavior of mode shapes in the cantilever beam. Due to the special properties of carbon nano tubes, by changing the ratio of them in the structure, flexibility and the amplitude of vibration change but at the same time the structural strength doesn't decrease even its flexibility increase. These features reduce the risk of sudden failures of mechanical vibrations, resonance and fatigue and also influence the damping behavior of laminate beam.

Keywords: Nano composite, CNT, Laminate, Natural frequency, Dynamic behavior

INTRODUCTION

Composite materials used in several ways and became replaced with different types of materials during these decades. One of the most ways of using them is in stressful and low weighted aerospace structures. These materials although have a good stiffness, specific hardness and but are light. Because of the sensibility and high risk of aerospace structures these composite materials must be chosen cautiously. The most reason for this increasing in using this kind of materials is carbon fibers.¹⁻⁴ Using CNTs (Carbon Nano tubes) achieved high flexibility, stiffness, specific hardness, specific stiffness and specific hardness per weight for composites. One of the important parts of manufacturing nano composite materials is scattering the nano materials like CNTs in the matrix because in this way the final material has a good performance⁵⁻¹⁴.

Another benefit of nano composite materials is low specific weight. It means that the material has high specific stiffness and high specific modulus defines as follow that ρ shows the density, E shows Young modulus and σ_{ult} shows the ultimate stiffness.⁵⁻¹⁴

$$\text{Specific stiffness} = \frac{\sigma_{ult}}{\rho}$$

$$\text{Specific modulus} = \frac{E}{\rho}$$

As the fiber length is bigger, the ability of add and orient them became easier. The orientation of fibers causes the higher stiffness in that direction. Epoxy is the most useable matrixes in composite materials in aerospace industry because they have high resistance.⁴⁻¹⁴

When high performance, stiffness and life time needed nano composites are good choices to use. The places with high tension and erosion, the parts with high impact are other places that nano composites can become useful⁵⁻¹⁴.

Fibers and reinforcements have some important factors to influence the laminate operation as a) length of fibers, b) the direction and orientation of fibers, c) scattering of fibers and reinforcements particles in matrixes like epoxy and, d) the material of fibers and matrixes¹⁻³. In structural dynamic analysis, knowledge of structure behavior in elastic zone is important. As an example, decreasing in flexibility causes higher stiffness and increases the natural frequency in structures and vice versa¹⁵. Change in the weight ratio and orientations of carbon fibers and CNTs are influenced on natural frequency of the main structure¹⁶⁻²³.

In laminates and composite materials the density and mechanical parameters of each lamina, according to weight ratio of the compounds

(epoxy as matrix, carbon fibers and CNTs as reinforcements) are able to calculate^{1-3,8-10, and 23}. According to displacements in each lamina and stiffness matrixes of them, by using LaGrange equations for the whole laminate, stiffness matrixes and displacement are calculated^{1-3,8-10}.

Due to the special properties of carbon nanotubes, by increasing the ratio of them in structure, flexibility and the amplitude of vibration are changed, but at the same time, the structural strength doesn't decrease even its flexibility increase. The damping behavior of structures is also affected by these changes¹⁶⁻²³.

In this paper the laminate composite materials which reinforced with carbon fibers are used. Then CNTs are replaced with these fibers at different volume ratios. Also the stiffness matrixes are calculated. Specific cantilever laminate beams are modeled and analysis in two methods with analytical and Abaqus software for bending vibration. The results from software shows little difference between them and results from analytical method.

The influence of volume percentage and orientation of CNTs in the cantilever laminate beam with orthogonal layers are numerically analyzed. The results show that how change in volume ratio and orientation of compounds are changed the natural frequency and vibration behavior of mode shapes in the cantilever beam.

MATEMATCAL AND METHODS

In this section, calculation of the micro and macro mechanical parameters of lamina and assembled laminate are described. The method based on computational logic in order to calculate the eigen values and eigenvectors are expressed. Ratio between reinforcements and matrixes can be expressed by weight ratio which uses during manufacturing process, or by volume ratio which uses in micro mechanical calculations¹⁻³. The index f is used to describe the fiber. The index m is used to describe the matrix and index c is used to describe the composite.

If v_f describe the volume ratio of fibers in composite and v_m describe the volume ratio of matrix, then it is obviously obtained that^{1-3,8-10}

$$v_m + v_f = 1 \quad (1)$$

According to above indices, the density of whole composite or lamina is calculated as follow^{1-3,8-10}

$$\rho_c = \rho_f v_f + \rho_m v_m \quad (2)$$

According to equation (3)¹⁻³, the young modulus in fiber direction (E_{11}), can be calculated as follows, while assuming that, material behavior is linear, and fibers and resins are connected completely

$$E_c = E_f v_f + E_m v_m \quad (3)$$

If more than one kind of fiber has been used, equation (4) should be used to calculate young modulus along fiber direction (E_{11})^{1-3,8-10}.

$$E_{11} = E_m v_m + E_{f1} v_{f1} + E_{f2} v_{f2} + \dots \quad (4)$$

In perpendicular direction of the fibers, at the lamina's section, the young modulus of this direction (E_{22}) is calculated from equation (5) as follows^{1-3, 8-10}.

$$\frac{1}{E_{22}} = \frac{v_f}{E_f} + \frac{v_m}{E_m} \quad (5)$$

But according to experimental results, equation (5) is changed to equation (6) with following parameters^{1-3, 23}.

$$\frac{E_{22}}{E_m} = \frac{1 + \eta \xi v_f}{1 - \eta v_f} \quad (6)$$

Where for fibers with circular sections, $\xi = 2(\xi \text{ reinforcing factor})$ and

$$\eta = \frac{(E_f/E_m) - 1}{(E_f/E_m) + \xi} \quad (7)$$

With previous assumptions, for shear modulus (G_{12}) equation (8) is used as below^{1-3, 8-10}.

$$\frac{1}{G_{12}} = \frac{v_f}{G_f} + \frac{v_m}{G_m} \quad (8)$$

With experimental results, equation (8) is changed to equation (9) as follow^{1-3, 23}.

$$\frac{G_{12}}{G_m} = \frac{1 + \eta \xi v_f}{1 - \eta v_f} \quad (9)$$

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Where for fibers with circular sections $\xi = 1$ (ξ reinforcing factor) and

$$\eta = \frac{(G_f/G_m)-1}{(G_f/G_m)+\xi} \quad (10)$$

Poisson coefficient is calculated by¹⁻³

$$\vartheta_{12} = \vartheta_f v_f + \vartheta_m v_m \quad (11)$$

The Poisson coefficients are different in plane of lamina¹⁻³, so

$$\vartheta_{21} = \vartheta_{12} \times \frac{E_{22}}{E_{11}} \quad (12)$$

When CNTs are added to the resin with certain volume ratio (v_{cnt}), equations (1) to (12) influenced.

The stiffness matrix for each lamina is calculated with results from above equations, so this matrix is shown as follow^{1-3,23}.

$$[Q] = \begin{bmatrix} \frac{E_{11}}{1-\vartheta_{12}\vartheta_{21}} & \frac{\vartheta_{21}E_{11}}{1-\vartheta_{12}\vartheta_{21}} & 0 \\ \frac{\vartheta_{12}E_{22}}{1-\vartheta_{12}\vartheta_{21}} & \frac{E_{22}}{1-\vartheta_{12}\vartheta_{21}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \quad (13)$$

Equation (13) was only for fibers that added in direction with $\theta=0$ (according to Figure 1). So if the orientation change the correct form of above equations can be written as follow^{1-3, 8-10}. If $n=\sin\theta$ and $m=\cos\theta$, then the matrix T was defined as equation (14).

$$[T] = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & (m^2 - n^2) \end{bmatrix} \quad (14)$$

According to Figure 1, tensions along axis 1&2 and x&y can be converted to each other with equation (15)¹⁻³.

$$\sigma_{12} = T\sigma_{xy} \quad (15)$$

In axis x&y the relation between stress and strain is shown as follow¹⁻³.

$$\sigma_{xy} = \bar{Q}\varepsilon_{xy} \quad (16)$$

That \bar{Q} can be calculated by set of equations in equation (17) by defined parameters¹⁻³.

$$\begin{aligned} \bar{Q}_{11} &= Q_{11}m^4 + 2(Q_{12} + 2Q_{33})n^2m^2 + Q_{22}n^4 \\ \bar{Q}_{22} &= Q_{11}n^4 + 2(Q_{12} + 2Q_{33})n^2m^2 + Q_{22}m^4 \\ \bar{Q}_{12} &= (Q_{11} + Q_{12} - 4Q_{33})n^2m^2 + Q_{12}(m^4 + n^4) \\ \bar{Q}_{33} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{33})n^2m^2 + Q_{33}(m^4 + n^4) \\ \bar{Q}_{13} &= (Q_{11} - Q_{12} - 2Q_{33})m^3n + (Q_{12} - Q_{22} + 2Q_{33})n^3m \\ \bar{Q}_{23} &= (Q_{11} - Q_{12} - 2Q_{33})n^3m + (Q_{12} - Q_{22} + 2Q_{33})m^3n \end{aligned} \quad (17)$$

According to plane stress analysis, the displacement in x, y and z directions are named u , v and w respectively. So the displacement field is calculated with equation (18)^{1-3, 16}.

$$u = \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \left\{ -z \frac{\partial}{\partial x} \quad -z \frac{\partial}{\partial y} \quad 1 \right\}^T w = \tilde{L}w \quad (18)$$

Strain can be calculated in equation (19) as follow¹⁶.

$$\varepsilon_p = \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \left\{ -\frac{\partial^2}{\partial x^2} \quad -\frac{\partial^2}{\partial y^2} \quad -2\frac{\partial^2}{\partial x \partial y} \right\}^T w = Lw \quad (19)$$

To obtain dynamic equations for free vibration, Lagrangian equations is used as follow¹⁶.

$$\frac{d}{dx} \left\{ \frac{\partial \theta}{\partial w} \right\} - \left\{ \frac{\partial \theta}{\partial w} \right\} = \{0\} \quad (20)$$

In equation (20), $\theta = T - \Pi_p$ and is called Lagrangian function, T is called kinetic energy and Π_p is called potential energy and are calculated as follow¹⁶.

$$T = \frac{1}{2} \int_V \rho \dot{u}^T \dot{u} dV \quad (21)$$

$$\Pi_p = \frac{1}{2} \int_V \varepsilon_p^T \sigma_p dV - \int_{S_\sigma} u^T \bar{t} dS - \int_V u^T b dV \quad (22)$$

In equation (22), \bar{t} is boundary force and b is volume force vector. By replacing the above equations in equation (20) the results is shown in equation (23)¹⁶.

$$\frac{d}{dx} \int_V \rho \frac{\partial}{\partial w} (\tilde{L}w)^T (\tilde{L}w) dV + \int_V \rho \frac{\partial}{\partial w} (Lw)^T Q(Lw) dV = \int_{S_\sigma} \rho \frac{\partial}{\partial w} (\tilde{L}w)^T \tilde{t} dS + \int_V \rho \frac{\partial}{\partial w} (\tilde{L})^T b dV \quad (23)$$

The eigenvalues are calculated from equation (31)^{16, 24}.

$$(K - \omega^2 M)\Omega = 0 \quad (31)$$

In the above equation, K is stiffness matrix which is came from assembling in the stiffness matrix of each element in equation (32)^{16, 24}.

$$K_{IJ} = \int_S L^T \phi_I Q L \phi_J dS \quad (32)$$

And M is assembled mass matrix according to equation (33)^{16, 24}.

$$M_{IJ} = \int_S \rho \tilde{L}^T \phi_I Q \tilde{L} \phi_J dS \quad (33)$$

EXPERIMENTAL AND SIMULATION

In this paper, different types of three layer laminates based on orientations setup include [0 90 0], [45 -45 45] and [90 0 90] are modeled. Each laminate is converted to a cantilever beam (see Figure 2) with 200mm in length (L), 50mm in wide (b) and 3mm in thickness (t), which each lamina has 1mm thickness and with boundary conditions that are shown in Figure 3. The laminas are as same as each other in ratios of combination parts that are shown in Table 1, and with mechanical properties which are shown in Table 2. In each lamina carbon fibers and CNTs are parallel. The results of bending vibration in present method are recorded and compared with Abaqus results in Tables 3 to 5 by their type of combination ratios and layers orientations.

The cantilever beams which modeled in Abaqus are meshed and shown in Figure 4. The Abaqus results for first ten natural frequencies with different ratios of CNTs are obtained and shown in Figures 5 to 7. According to the laminate's layers set up ([0 90 0], [45 -45 45], [90 0 90]) the mode shapes of these first ten natural frequencies are also obtained and recorded in Figures 8 to 10.

RESULTS AND DISCUSSIONS

By looking at the Table 3 which shows the first three natural frequencies in bending vibration, with lamina combination as [0 90 0], difference between present method and Abaqus results are acceptable. The mode shapes of these frequencies are also shown in Figure 8 (see numbers 1, 4 and 7 in Figure 8). More, from the Table 3 it can be seen that in this arrangement, at the first natural bending frequencies (see column 1 in Table 3), in 15% CNT's volume ratio the maximum value happened. The cause of this event could be CNT's special abilities and layers arrangements but in the other columns natural frequencies decrease by increasing in CNT's volume ratio.

By subtilizing in the Table 4 which shows the first three natural frequencies in bending vibration, with lamina combination as [90 0 90], difference between present method and Abaqus results are acceptable. The mode shapes of these frequencies are also shown in Figure 10 (see numbers 1, 3 and 5 in Figure 10). More from the above table it can be seen that in this arrangement, at the second natural bending frequencies (see column 2 in Table 4), in 10% of CNT's volume ratio and also in third natural bending frequencies (see column 3 in Table 4), in 15% of CNT's volume ratio, maximum values happened but in first column natural frequency increase by increasing in CNT's volume ratio.

According to Table 5 which shows the first three natural frequencies in bending vibration, with lamina combination as [0 90 0], difference between present method and Abaqus results are acceptable. The mode shapes of these frequencies are also shown in Figure 9 (see numbers 1, 3 and 7 in Figure 9). More, from the Table 5, it can be seen that in this arrangement at the first and second natural bending frequencies (see columns 1 and 2 in Table 5), in 5% of CNT's volume ratio maximum values happened but in column 3 by increasing in volume ratio of CNT's the frequency value decrease.

The first bending natural frequencies in first column of Tables 3 to 5 show difference in results. It is obtained that in [0 90 0] laminate, natural frequencies have higher values beside with the others, because by this arrangements in layers and CNT's properties, the stiffness matrixes in this case have higher value than the others.

By verification Abaqus results with present method that are shown in Tables 3 to 5, the ten first natural frequencies are recorded in Figures 5 to 7 by the variety of CNT's volume ratios and frequency numbers. All of the ten first mode shapes are shown in Figures 8 to 10 respectively, by ascending their frequency numbers and layers combinations for 15% CNT's volume ratio.

As shown in Figure 5, by increasing in CNT's volume ratio some of frequency values are increase and some of them are decrease. Also in certain volume ratios of CNT, with increasing in frequency numbers, the frequency value increase. These changes are analogous in Figures6 and 7 too.

According to Figures8 to 10 the modes shapes are not in same vibration as each other in simultaneously modes. For example according to layers combination in [0 90 0] the second bending mode happens in 4th mode but in [90 0 90] it happens in third and so in [45 -45 45]. Also it shows that, in first ten mode shapes, only in [90 0 90], four kind of bending vibration happens but in the others only three bending vibration got happen.

According to our obtained result from Abaqus, with different volume ratios of CNT the displacements of cantilever beam are changed in certain mode shapes. In other words with changing in stiffness matrix in certain orientation, frequency values are changed and also the displacements of those certain nodes changed.

FIGURES AND TABLES

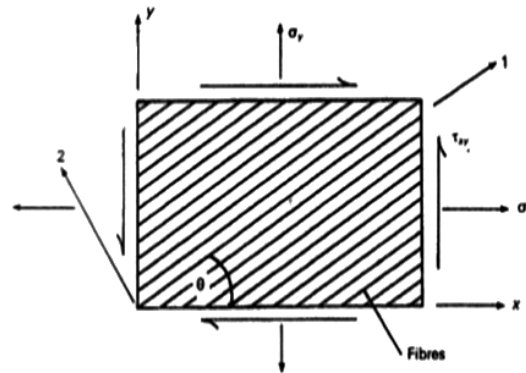


Figure 1.Lamina in rectangular x-y datum and orientation of fibers.^{1,2}

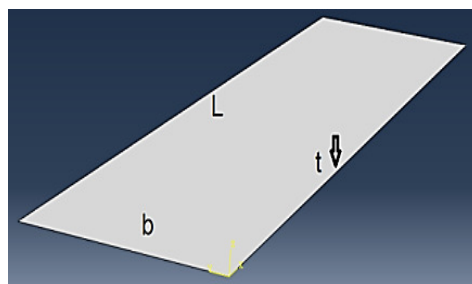


Figure 2.Laminate beam with three layers simulated in Abaqus with shell element and define parameters.

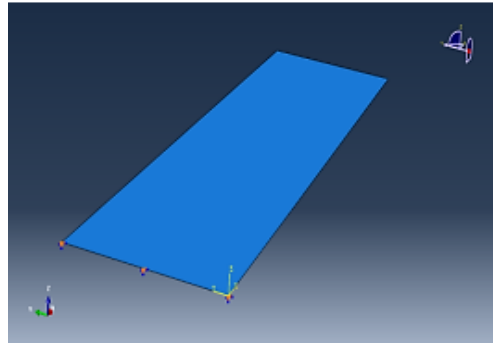


Figure 3.Boundary conditions apply on laminate the cantilever beam.

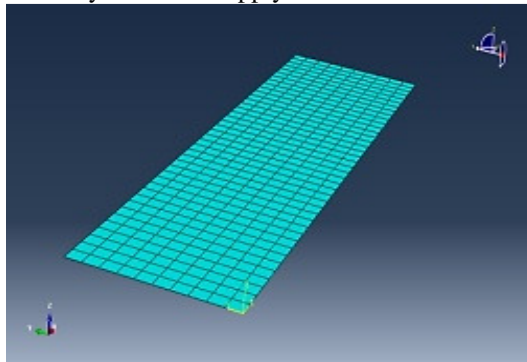


Figure 4.Laminate beam meshed in Abaqus software.

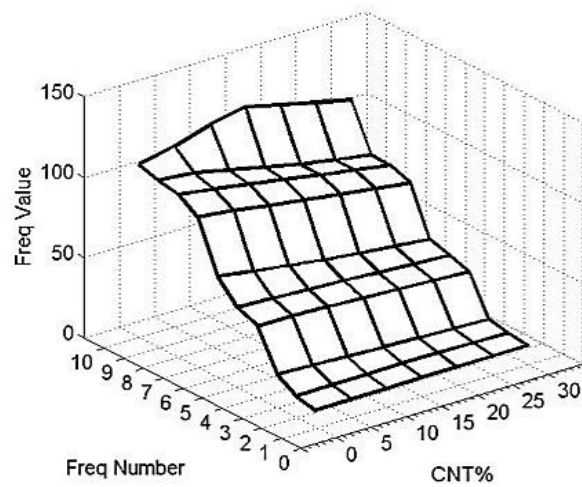


Figure 5.First ten natural frequency value (Hz) of three layer laminate beam with combination [0 90 0].

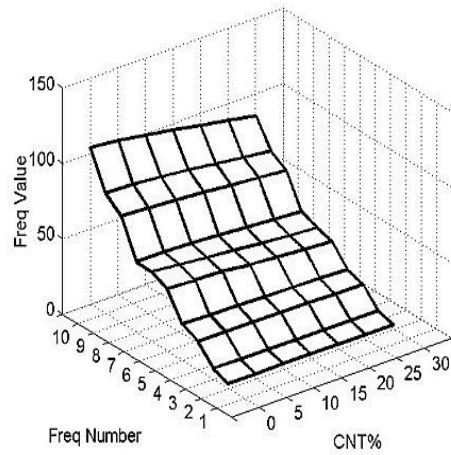


Figure 6.First ten natural frequency value (Hz) of three layer laminate beam with combination [45 -45 45].

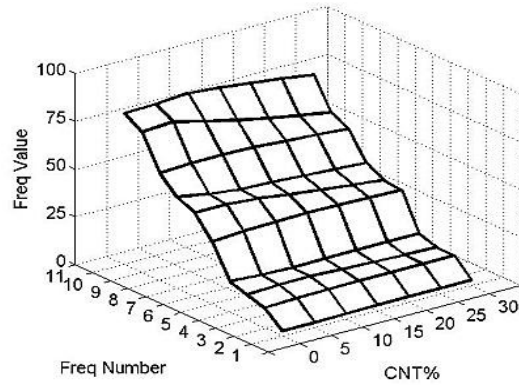
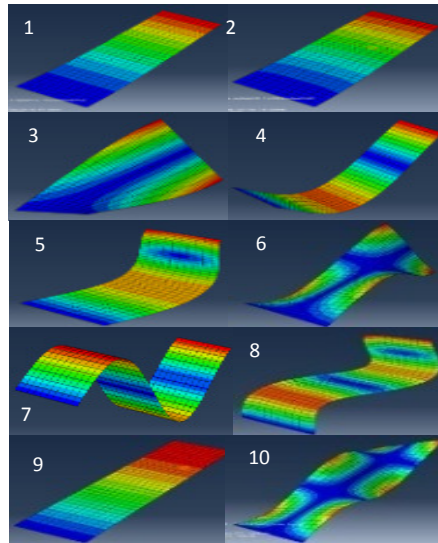
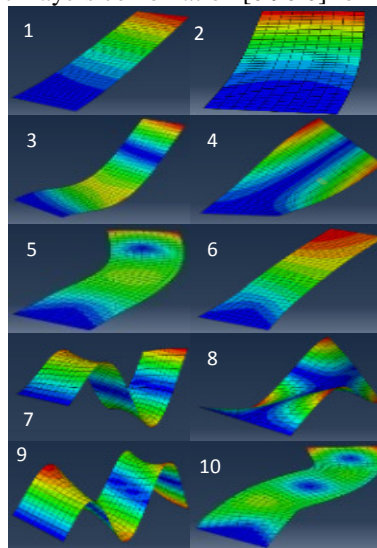


Figure 7.First ten natural frequency value (Hz) of three layer laminate beam with combination [90 0 90].



[0 90 0]

Figure 8.First 10 mode shape of the laminate with layers combination [0 90 0] for 15% CNT's volume ratio.



[45 -45 45]

Figure 9.First 10 mode shape of the laminate with combination [45 -45 45] for 15% CNT's volume ratio.

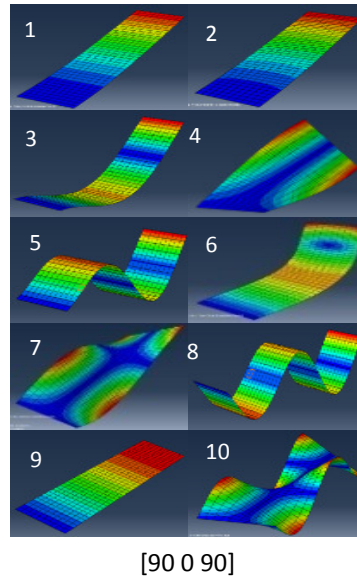


Figure 10. First 10 mode shape of the laminate with combination [90 0 90] for 15% CNT's volume ratio.

Model number	$V_{CNT} \%$	$V_f \%$	$V_m \%$
1	0	65	35
2	5	55	35
3	10	50	35
4	15	45	35
5	20	40	35
6	25	35	35
7	30	30	35

Table 1. Volume percent of combination parts in each lamina.

Material	E_{11} (Pa)	E_{22} (Pa)	G_{12} (Pa)	ν_{12}	ρ (Kg/m ³)
Epoxy Matrix	3.40E+09	3.40E+09	1.30E+09	0.3	1.28E+03
Carbon Fiber	2.30E+11	2.20E+10	2.20E+10	0.3	1.78E+03
CNT	1.00E+12	1.10E+10	3.50E+10	0.3	1.80E+03

Table 2. Mechanical properties which are used for modeling the laminas.^{1,3, 5, 6, 10}

$V\%_{CNT}$	First bending mode freq. (Hz) with present method	First bending mode freq. (Hz) with Abaqus	Second bending mode freq. (Hz) with present method	Second bending mode freq. (Hz) with Abaqus	Third bending mode freq. (Hz) with present method	Third bending mode freq. (Hz) with Abaqus
0	10.88	10.701	44.63	44.599	93.84	93.77

5	11.21	11.194	44.16	44.149	91.26	91.127
10	11.53	11.486	43.38	43.317	88.11	88.036
15	11.79	11.741	42.52	42.433	85.05	84.891
20	11.72	11.7	41.31	41.227	81.77	81.622
25	11.68	11.666	40.24	40.103	78.66	78.507
30	11.62	11.583	40.15	39.001	75.81	75.542

Table 3. The natural frequencies obtained from the two methods in the first three bending modes of vibration from laminate with [0 90 0].

V% CNT	First bending mode freq. (Hz) with present method	First bending mode freq. (Hz) with Abaqus	Second bending mode freq. (Hz) with present method	Second bending mode freq. (Hz) with Abaqus	Third bending mode freq. (Hz) with present method	Third bending mode freq. (Hz) with Abaqus
0	2.393	2.46	13.32	13.179	32.78	32.645
5	2.5093	2.58	13.93	13.88	33.91	33.7
10	2.6541	2.71	14.27	14.133	34.25	34.179
15	2.8103	2.92	13.44	13.35	34.64	34.512
20	2.8744	2.98	12.72	12.688	34.19	34.132
25	2.9541	3.13	12.11	12.07	33.82	33.776
30	3.0239	3.24	11.63	11.51	33.39	33.32

Table 4. The natural frequencies obtained from the two methods in the first three bending modes of vibration from laminate with [90 0 90].

V% CNT	First bending mode freq. (Hz) with present method	First bending mode freq. (Hz) with Abaqus	Second bending mode freq. (Hz) with present method	Second bending mode freq. (Hz) with Abaqus	Third bending mode freq. (Hz) with present method	Third bending mode freq. (Hz) with Abaqus
0	3.41	3.3955	18.84	18.815	46.69	46.63
5	3.43	3.4061	18.91	18.87	46.48	46.416
10	3.39	3.3771	18.77	18.672	45.73	45.642
15	3.36	3.3389	18.42	18.391	44.81	44.689
20	3.32	3.2607	18.02	17.9	43.45	43.342
25	3.23	3.1864	17.88	17.422	42.32	42.033
30	3.18	3.1095	17.11	16.932	40.97	40.722

Table 5. The natural frequencies obtained from the two methods in the first three bending modes of vibration from laminate with [45 -45 45].

CONCLUSION

Composite materials especially carbon epoxy reinforced with carbon fibers are used in several ways. Using carbon nano tubes achieved high performance and specific parameters per weight for composites. When high performance, stiffness and life time needed, nano composites are good choices to use.

The results obtained show that changing in CNT's volume ratio effect mechanical properties, stiffness and mass matrix of the laminate. Therefore, these changes can affect the natural frequencies of structures.

In Tables 3 to 5 the difference between present method and Abaqus results are acceptable, so the Abaqus model use for other kind on analysis results such as mode shapes behavior, all first ten natural frequencies and nodal displacements.

By changing in CNT's volume ratio, the stiffness matrix effected. So any changes in stiffness matrix cause change in natural frequency and displacements in nodes. It is also concluded that in some mode shapes, the natural frequency value increase by increasing the CNT's volume ratio but in some others it decrease. Based on the results it can be seen that the arrangement of layers in the laminate effect the stiffness matrix of the beam. So the mode shapes of the vibration are changed.

With the addition of CNTs and laminates layers set up, hardness is changed thereby change in flexibility that can change the natural frequencies is observed. Due to the special properties of carbon nanotubes, by increasing the ratio of them in structure, flexibility and the amplitude of vibration change, but at the same time the structural strength doesn't decrease even its flexibility increase. This feature reduces the risk of sudden failures of mechanical vibrations and fatigue and also can change the damping properties and behaviour.

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