THE EXPERIMENT
INTERNATIONAL JOURNAL OF SCIENCE AND TECHNOLOGY

ISSN-2319-2119

REVIEW ARTICLE

Bharat Singh et al, The *Experiment, April, 2013 Vol. 9(1), 526-531*

FIXED POINT THEOREM IN FUZZY METRIC SPACE BY USING OCCASIONALLY WEAKLY COMPATIBLE MAPS

ABSTRACT

In this paper we have generalized the result of Kamel Wadhwa and Hariom Dubey [1] by using Occasionally Weakly compatible Maps.

INTRODUCTION

Fuzzy Set was introduced and defined by Zadeh [4]. Kramosil and Michalek [5] introduced fuzzy metric space,George and Veeramani [6] modified the notion of fuzzy metric space with the help of continuous t-norm. Vasuki [7] proved fixed point theorem for R-weakly commuting mapping.Pant [8,9] introduced the new concept of common fixed point theorems. The concept of compatible maps introducede by Kramosil and Michalek [5] and weakly compatible maps in fuzzy metric space is generalized by A. Al. Thagafi and Nasser Shahzad [10] by introducing the concept of occasionally weakly compatible mappings.

PRELIMINARIES

Definition 2.1 [11] A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is continuous t-norm if $*$ satisfies the following conditions:

(i) $*$ is commutative and associative;
(ii) $*$ is continuous;
(iii) $a * 1 = a$ for all $a \in [0,1]$;
(iv) $a * b \le c * d$ where $a \le c$ *and* $b \le d$ for all $a, b, c, d \in [0,1]$

Definition 2.2 [6] A 3-tuple $(X, M,*)$ is said to be a fuzzy metric space if X is an arbitrary set $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0,\infty)$ satisfying the following condtions,$\forall\ x, y, z \in X, s, t > 0$,

(f1) $M(x, y, t) > 0$;

(f2) $M(x, y, t) = 1$, *if and only if* $x = y$.

(f3) $M(x, y, t) = M(y, x, t)$;

(f4) $M(x, y, t) * M(y, z, s) \le M(x, z, t + s)$;

(f5) $M(x, y, .):(0,\infty) \to (0,1]$ is continuous.

Then M is called a fuzzy metric on X .Then $M(x, y, t)$ denotes the degree of nearness between x and y with respect to t .

Example 2.3 (($Induced\ fuzzy\ metric$ [**6**]) Let (X, d) be metric space . Denote $a * b = ab\ for\ all\ a, b \in [0,1]$ and let M_d be fuzzy sets on $X^2 \times (0,\infty)$ defined as follows;

$$M_d(x, y, t) = \frac{t}{t + d(x, y)}$$

Then $(X, M_d,*)$ is a fuzzy metric space. We call this fuzzy metric induced by a metric d as the standard intuitionistic fuzzy metric .

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Definition 2.4 [12] Two self mappings f and g of a fuzzy metric space $(X, M,*)$ are called compatible if $\lim_{n\to\infty} M(fgx_{n},gfx_n, t) = 1$ whenever$\{x_n\}$ is a sequence in X such that

$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x$ for some $x \in X$.

Definition 2.5 Let A and B be two self maps of X. A point $x \in X$ is called a coincidence point of A and B iff Ax = Bx. We Call Ax = Bx = w a point of coincidence of A and B.

Definition 2.6 Two self maps A and B are called weakly compatible if they commute at coincidence point.

Definition 2.7 Two self maps A and B are called occasionally weakly compatible if there is a

point $x \in X$ which is a co incidence point of A and B at which A and B commute .

Lemma 2.8 Let $(X, M,*)$ be fuzzy metric space if there exist $k \in (0,1)$ such that $M(x, y, kt) \geq M(x, y, t)$ for all $x, y \in X$ *and* $t > 0$, *then* $x = y$.

Lemma 2.9 Let $(X, M,*)$ be fuzzy metric space and for all $x, y \in X$ *and* $t > 0$ if there exist $k > 1$ *such that* $M(x, y, kt) \leq M(x, y, t)$, *Then* $x = y$.

Lemma 2.10 Let X be a set A and B owc self maps of X. if A and B have a unique point of councidence w = Ax = Bx , then w is the unique common fixed point of A and B.

3.Main Result

Theorem 3.1

Let $(X, M,*)$ the be complete fuzzy metric space and let A , B , S , T be self mapping of X Let the pairs (A, S) *and* (B, T) *be* *OWC* and k$>$ 1 then

$$M(Ax, By, kt) \leq min\left\{\begin{matrix} M(Sx, Ty, t), M(Sx, Ax, t), M(Ty, By, t), M(Sx, By, t), M(Ty, Ax, t), \\ \dfrac{aM(Ax, Ty, t) + b(By, Sx,, t) + cM(Sx, Ty, t)}{a+b+c}. \dfrac{1 + M(Ax, Sx, t)}{2} \end{matrix}\right\}$$

….(1) for all $x, y \in X$ *and* $t > 0$ such that $Aw = Sw = w$ and a unique point $z \in X$ such that

$Bz = Tz = z$ .Moreover $z = w$, so that there is a unique common fixed point of A,B,S and T.

Proof : Let the pairs (A, S) *and* (B, T) are *OWC* so there are points $x, y \in X$ such that Ax = Sx and By = Ty we claim that Ax = By. If not then by inequality (1)

$$M(Ax, By, kt) \leq min\left\{\begin{matrix} M(Sx, Ty, t), M(Sx, Ax, t), M(Ty, By, t), M(Sx, By, t), M(Ty, Ax, t), \\ \dfrac{aM(Ax, Ty, t) + b(By, Sx,, t) + cM(Sx, Ty, t)}{a+b+c}. \dfrac{1 + M(Ax, Sx, t)}{2} \end{matrix}\right\}$$

$$M(Ax, By, kt) \leq min\{M(Ax, Ty, t), 1,1, M(Ax, By, t), M(By, Ax, t), 1\}$$

ISSN-2319-2119

REVIEW ARTICLE

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$M(Ax,By,kt) \leq M(Ax,By,t)$ Then by lemma (2.9)

Ax = By.

Suppose that there is another point z s.t. Az = Sz.Then by inequality (1) we have $Az = Sz = By = Ty$ so $Ax = Az$ and $w = Ax = Sx$ is the unique point of coincidence of A and S. By lemma 2.10 w is the only common point of A and S. Similiarly there is a unique point $z \in X$ s.t. z = Bz = Tz

Assume that $w \neq z$ then by (1)

$$M(w,z,kt) = M(Aw,Bz,kt)$$

$$M(Ax,By,kt) \leq min\left\{\begin{matrix} M(Sx,Ty,t), M(Sx,Ax,t), M(Ty,By,t), M(Sx,By,t), M(Ty,Ax,t), \\ \frac{aM(Ax,Ty,t) + b(By,Sx,,t) + cM(Sx,Ty,t)}{a+b+c}. \frac{1 + M(Ax,Sx,t)}{2} \end{matrix}\right\}$$

$$M(Aw,Bz,kt) \leq min\left\{\begin{matrix} M(Sw,Tz,t), M(Sw,Aw,t), M(Tz,Bz,t), M(Sw,Bz,t), M(Tz,Aw,t), \\ \frac{aM(Aw,Tz,t) + b(Bz,Sw,,t) + cM(Sw,Tz,t)}{a+b+c}. \frac{1 + M(Aw,Sw,t)}{2} \end{matrix}\right\}$$

$$M(w,z,kt) \leq min\left\{M(w,z,t), 1,1, M(Aw,Bz,t), M(Bz,Aw,t), 1\right\}$$

$$M(w,z,kt) \leq M(w,z,t)$$

then by lemma (2.9)

Therefore w =z . z is a common fixed point of A, B, S and T.

Uniqueness:

Let u be another common fixed point of A , B , S and T. Then put x = z and y = u in (1)

$$M(Az,Bu,kt) \leq min\left\{\begin{matrix} M(Sz,Tu,t), M(Sz,Az,t), M(Tu,Bu,t), M(Sz,Bu,t), M(Tu,Az,t), \\ \frac{aM(Az,Tu,t) + b(Bu,Sz,,t) + cM(Sz,Tu,t)}{a+b+c}. \frac{1 + M(Az,Sz,t)}{2} \end{matrix}\right\}$$

$$M(z,u,kt) \leq max\left\{\begin{matrix} M(z,u,t), 1,1, M(z,u,t), M(u,z,t), \\ 1 \end{matrix}\right\}$$

$$M(z,u,kt) \leq M(z,u,t)$$

Then by lemma (2.9) z =u

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Theorem 3. 2

Let $(X, M, *)$ the be complete fuzzy metric space and let A , B , S , T be self mapping of X Let the pairs (A, S) *and* (B, T) *be* *OWC* and $k > 1$ and $\propto + \beta = 1$ then

$$M(Ax, By, kt) \leq min \left\{ M(Sx, Ty, t), M(Sx, Ax, t), M(Ty, By, t), M(Sx, By, t), M(Ty, Ax, t), \{\propto (Sx, By, t) + \beta \, min\{M(By, Ty, t), M(Sx, Ax, t), M(Sx, Ty, t)\}\} \right\}$$

……..(2)

for all $x, y \in X$ *and* $t > 0$ such that $Aw = Sw = w$ and a unique point $z \in X$ such that

$Bz = Tz = z$ .Moreover $z = w$, so that there is a unique common fixed point of A , B , S and T.

Proof: Let the pairs (A, S) *and* (B, T) are *OWC* so there are points $x, y \in X$ such that Ax = Sx and By = Ty we claim that Ax = By. If not then by inequality (2)

$$M(Ax, By, kt) \leq min \left\{ M(Ax, By, t), M(Ax, Ax, t), M(By, By, t), M(Ax, By, t), M(By, Ax, t), \{\propto (Sx, By, t) + \beta min\{M(By, By, t), M(Ax, Ax, t), M(Ax, By, t)\}\} \right\}$$

$$M(Ax, By, kt) \leq min \left\{ M(Ax, By, t), 1, 1, M(Ax, By, t), M(By, Ax, t), \{\propto (Ax, By, t) + \beta min\{1, 1, M(Ax, By, t)\}\} \right\}$$

$$M(Ax, By, kt) \leq M(Ax, By, t)$$

Then by lemma (2.9)

Ax = By.

Suppose that there is another point z s.t. Az = Sz.Then by inequality (2) we have $Az = Sz = By = Ty$ so $Ax = Az$ and $w = Ax = Sx$ is the unique point of coincidence of A and S. By lemma 2.10 w is the only common point of A and S. Similiarly there is a unique point $z \in X$ s.t. z = Bz = Tz

Assume that $w \neq z$ then by (2)

$$M(w, z, kt) = M(Aw, Bz, kt)$$

$$M(Ax, By, kt) \leq min \left\{ M(Sx, Ty, t), M(Sx, Ax, t), M(Ty, By, t), M(Sx, By, t), M(Ty, Ax, t), \{\propto (Ax, By, t) + \beta \, min\, \{M(By, Ty, t), M(Sx, Ax, t), M(Sx, Ty, t)\}\} \right\}$$

Put x = w and y = z in inequality (2)

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$$M(Aw,Bz,kt) \le min\left\{M(Sw,Tz,t), M(Sw,Aw,t), M(Tz,Bz,t), M(Sw,Bz,t), M(Tz,Aw,t), \{\propto (Sw,Bz,t) + \beta\ min\ \{M(Bz,Tz,t), M(Sw,Aw,t), M(Sw,Tz,t)\}\}\right\}$$

$$M(w,z,kt) \le min\ \left\{M(w,z,t), M(w,w,t), M(z,z,t), M(w,z,t), M(z,w,t), \{\propto (w,z,t) + \beta min\ \{M(z,z,t), M(w,w,t), M(w,z,t)\}\}\right\}$$

$$M(w,z,kt) \le min\left\{M(w,z,t), 1,1, M(w,z,t), M(z,w,t), \{\propto (w,z,t) + \beta min\{1,1, M(w,z,t)\}\}\right\}$$

$$M(w,z,kt) \le M(w,z,t)$$

Then by lemma (2.9)

w = z

Uniqueness:

Let u be another common fixed point of A,B,S and T.Then put x = z and y= u in (2)

$$M(Ax,By,kt) \le min\ \left\{M(Sx,Ty,t), M(Sx,Ax,t), M(Ty,By,t), M(Sx,By,t), M(Ty,Ax,t), \{\propto (Sx,By,t) + \beta min\ \{M(By,Ty,t), M(Sx,Ax,t), M(Sx,Ty,t)\}\}\right\}$$

$$M(Az,Bu,kt) \le min\ \left\{M(Sz,Tu,t), M(Sz,Az,t), M(Tu,Bu,t), M(Sz,Bu,t), M(Tu,Az,t), \{\propto (Sz,Bu,t) + \beta min\ \{M(Bu,Tu,t), M(Sz,Az,t), M(Sz,Tu,t)\}\}\right\}$$

$$M(z,u,kt) \le min\ \left\{M(z,u,t), M(z,z,t), M(u,u,t), M(z,u,t), M(u,z,t), \{\propto (z,u,t) + \beta min\ \{M(u,u,t), M(z,z,t), M(z,u,t)\}\}\right\}$$

$$M(z,u,kt) \le min\ \left\{M(z,u,t), 1,1, M(z,u,t), M(u,z,t), \{\propto (z,u,t) + \beta min\ \{1,1, M(z,u,t)\}\}\right\}$$

$$M(z,u,kt) \le M(z,u,t)$$

Then by lemma (2.9)

z = u

INTERNATIONAL JOURNAL OF SCIENCE AND TECHNOLOGY

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REVIEW ARTICLE

Bharat Singh et al, The *Experiment, April, 2013 Vol. 9(1), 526-531*

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