

## EXPERIMENTAL VERIFICATION OF COULOMB'S LAW IN MAGNETISM USING THE CONCEPT OF STATIC FRICTION

### ABSTRACT

We have designed an experiment to verify the Coulomb's law of magnetic monopole attraction using the concept of static friction both in horizontal and inclined surfaces. Although the concept of monopole has been put down still we can verify the law by adding some correction terms to it which arise due to the attraction and repulsion of other nearby poles.

Charles Augustin de Coulomb, in 1777, showed that the forces of attraction or repulsion between electrically charged bodies and between magnetic monopoles (although presently the concept of magnetic monopole has been discarded because of lack of experimental verification still for computational purpose we use the term *magnetic pole strength*) obey an inverse square law like that derived for gravity by Newton. The law related to magnetic force may be stated as [1]

$$F_m = \frac{\mu_0}{4\pi} \frac{p_1 p_2}{d^2}, \quad (1)$$

where  $p_1$  and  $p_2$  are the pole strengths of the monopoles situated  $d$  distance apart and  $\mu_0$  being a constant depending upon the intervening medium, namely magnetic permeability. To make the measurements necessary for this, Coulomb and John Michell (in 1785) invented torsion balance.

In our present work we suggest a simple experimental verification of Coulomb's law in magnetism [1], provided we consider the monopoles do exist for computational purposes as mentioned earlier. The necessary steps of the experiment and relevant theories are explained below.

**STEP 1:** A small piece of bar magnet (*magnet 1*) is put on a white paper. It is placed in magnetic meridian (N-pole facing magnetic north direction) in the usual way with the help of a magnetic needle. Lines of forces are drawn on the paper with the help of the magnetic needle. We do not need to draw a large number of those lines. Only a few lines starting (finishing) from (to) the N (S) pole will be sufficient (as shown in fig.1). Also we have to identify the neutral points P, P' (as shown in fig.1) corresponding to the bar magnet in particular and the earth's horizontal component of the magnetic field at that place with the help of the magnetic needle in the usual manner (by slowly moving the needle from the neutral region of the magnet and identifying the points of no net force on both sides of the magnet). After removing the magnet now if we extrapolate the lines of forces into the position of the magnet we may identify the exact locations of the magnetic poles (as shown in fig.1).

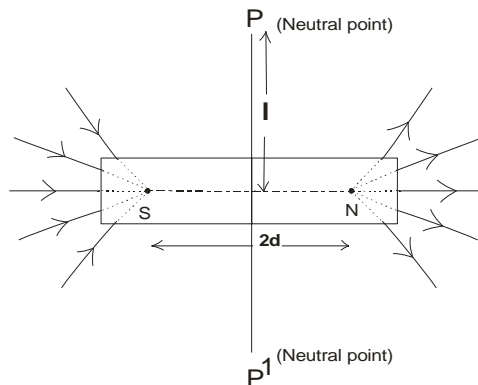


Fig.1: Extrapolating the lines of forces into the magnet (placed in magnetic meridian) we have determined the position of the

poles as shown by the dotted lines.  $P$  and  $P^1$  are the neutral points, which lie on the perpendicular bi-sector of the magnet at a distance  $l$  and  $l'$  respectively from the magnetic axis. Magnetic length of the magnet being  $2d$ .

Let the distances of the N and S poles from the respective ends of the magnet are  $L_1$  and  $L_1'$  (which can be measured easily with a simple ruler) respectively. Usually they are almost identical. Measuring the geometric length of the bar magnet  $L_1^g$  we may easily verify the relation

$$\frac{L_1^g - L_1 - L_1'}{L_1^g} = 0.84. \quad (2)$$

**STEP 2:** Let the pole strength of magnet 1 be  $p_1$ . Hence the magnetic field at P (Gauss's *tan B* position) is given by [2]

$$B_p = \frac{\mu_0}{4\pi} \frac{2p_1 d_1}{(l_1^2 + d_1^2)^{3/2}}. \quad (3)$$

Here  $\mu_0$ ,  $2d_1$  and  $l_1$  are the permeability of free space, magnetic length NS of the bar magnet and the perpendicular distance of the neutral point from the magnet respectively. While  $\mu_0$  has a standard value ( $10^{-7} \text{ Hm}^{-1}$ ) the other two parameters can be measured with the help of a simple ruler.

By definition of neutral point,  $B_p = H_e$ ,  $H_e$  being the earth's horizontal component of magnetic field at P (which may be taken as  $3 \times 10^{-5} \text{ T}$ ). Hence we may have

$$p_1 = \frac{4\pi H_e (l_1^2 + d_1^2)^{3/2}}{\mu_0 \cdot 2d_1}. \quad (4)$$

**STEP 3:** The entire process has been repeated for the other neutral point  $P^1$  and again the pole strength  $p_1'$  has been obtained. The arithmetic mean of  $p_1$  and  $p_1'$  may be considered to be the mean pole strength for magnet 1. Hence

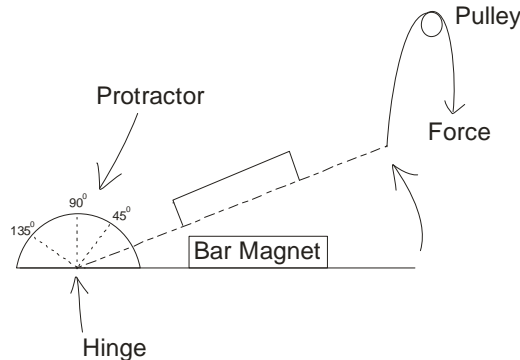
$$p_1'' = \frac{p_1 + p_1'}{2}. \quad (5)$$

**STEP 4:** All the previous steps has been repeated for another bar magnet (magnet 2) of slightly different dimensions (different length and width but having the same breadth so that the line joining the poles of the magnets must remain horizontal) and its pole strength has been determined in a similar manner as

$$P_2'' = \frac{P_2 + P_2'}{2} \quad (6)$$

**STEP 5:** Magnet 1 is now placed on a thick horizontal board (preferably made of ply) one end of which is hinged on the horizontal top of a table the other end is free to rotate along the circumference of a circle having center at the hinged end. The idea is explained in fig.2. A string hangs the open end of the board with the help of a pulley fixed above the board. By pulling the open end of the string passing over the pulley we can change the angular orientation of the board as much as we can. A protractor is fitted at the hinged base of the board vertically so as to measure the angular position of the board with respect to the horizontal direction. Using this arrangement we may carefully determine the angle of rest of magnet 1 on this board. Let that angle be  $\theta_1$ . Hence the coefficient of static friction between the magnet 1 and the board is [3]

$$\mu_1 = \tan \theta_1 \quad (7)$$



**Fig.2:** The bar magnet is placed on a horizontal rigid board one side of which is hinged on the table and the free side is attached to a string passing over a pulley. The hinged end is fitted with a protractor as shown in the figure. Applying force to the open end of the string the angle of rest for the system can be measured.

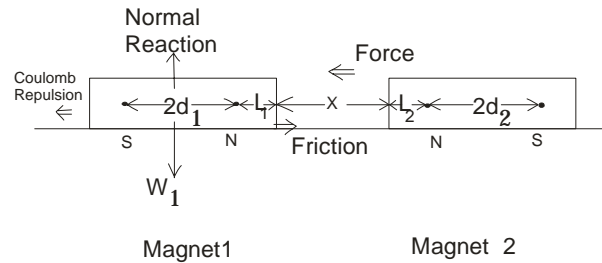
The angle is determined for different orientations and positions of the magnet on the incline. And finally the mean value must be taken.

**STEP 6:** The entire process has been repeated for the magnet 2 and we have obtained

$$\mu_2 = \tan \theta_2 \quad (8)$$

**STEP 7:** Now the board is placed horizontally with the help of the string and magnets are placed nearby along their lengths in such a way that both N poles are facing each other. The initial distance between the poles should be such that both the magnets are at rest. We now slowly displace magnet 2 towards magnet 1 (as shown in fig.3) and try to identify carefully the moment at which magnet 1 just starts its motion away from magnet 2 due to Coulomb repulsion. Let the distance between the magnet ends at this very moment be  $x_1$ , which we may term as the closest distance of approach for magnet 1. Noting that the distances corresponding to  $L_1$  and  $L_1'$  (related to magnet 1) are  $L_2$  and  $L_2'$  for magnet 2 we may obtain the expression for Coulomb repulsion between two N poles as

$$F_1^C = \frac{\mu_0}{4\pi} \frac{P_1 P_2}{(L_1 + x_1 + L_2)^2} \quad (9)$$



**Fig.3: Two magnets are placed on the horizontal board with their similar poles facing each other. As the net magnetic force between the above magnets is repulsive hence it is balanced by the static frictional forces between the magnet and the board.**

This force is now exactly counterbalanced by the limiting friction (proportional to normal reaction) between *magnet 1* and the board. Since the board is now horizontal the reaction applied on the magnet by the board will be exactly equal to its weight  $W_1$ , which may be determined by a simple or spring balance. Hence we may write the frictional force as [3]

$$F_1^f = \mu_1 W_1 \quad (10)$$

We may verify that  $F_1^C = F_1^f$ . The effect of force between far poles is not considered here. One may easily check that they will effect only slightly on the final form of Coulomb force and will be almost negligible if we choose sufficiently long magnets. The corresponding correction factors are discussed below.

1<sup>st</sup> order corrections: These arise because of the attraction between N (S) pole of *magnet 2* and S (N) pole of *magnet 1*. The correction term will obviously be

$$F_{Correction}^1 = \frac{\mu_0}{4\pi} \frac{P_1 P_2}{(2d_1 + L_1 + x_1 + L_2)^2} + \frac{\mu_0}{4\pi} \frac{P_1 P_2}{(L_1 + x_1 + L_2 + 2d_2)^2} \quad (11)$$

The above factor has to be subtracted from the right hand side of (9).

2<sup>nd</sup> order correction: This is much smaller than the previous correction term and arises because of the repulsion between the S poles of *magnet 1* and *magnet 2*. The correction factor will be

$$F_{Correction}^2 = \frac{\mu_0}{4\pi} \frac{P_1 P_2}{(2d_1 + L_1 + x_1 + L_2 + 2d_2)^2} \quad (12)$$

The above term should be added to the right hand side of (9).

The reason why in step 7 we choose the repelling force (by facing similar poles) instead of choosing attractive force (by facing opposite poles) is that the moment of repulsion can be better identified manually instead of attraction. As because in the earlier case it is moving away from us while in the latter it is moving towards us.

**STEP 8:** Step 7 has been repeated for magnet 2 now and we will obtain the closest distance of approach  $x_2$  for magnet 2. Now the magnetic force will be exactly counter balanced by the limiting frictional force between magnet 2 and the board, namely

$$F_2 = \mu_2 W_2. \quad (13)$$

**STEP 9:** Both step 7 and step 8 is repeated for S poles.

We now present some experimental results below.

**TABLE 1 (results corresponding to magnet 1)**

$W_1$ (Kg) $\times 10^{-2}$	$L_1^s$ (m) $\times 10^{-2}$	$2d_1$ (m) $\times 10^{-2}$	$\frac{l_1 + l_1' }{2}$ (m) $\times 10^{-2}$	$L_1$ (m) $\times 10^{-2}$	$P_1''$ (Am)	$\theta_1$	$\mu_1$	$x_1$ (m) $\times 10^{-2}$	$F_1^C$ (N) $\times 10^{-3}$	$F_1^I$ (N) $\times 10^{-3}$
3.156	5.4	4.3	14.55	0.55	22.198	$31^0$	0.601	1.1	105.5	185.9

$$F^1_{Correction} = 23.4 \times 10^{-3} N. \quad F^2_{Correction} = 4.2 \times 10^{-3} N.$$

**TABLE 2 (results corresponding to magnet 2)**

$W_2$ (Kg) $\times 10^{-2}$	$L_2^s$ (m) $\times 10^{-2}$	$2d_2$ (m) $\times 10^{-2}$	$\frac{l_2 + l_2' }{2}$ (m) $\times 10^{-2}$	$L_2$ (m) $\times 10^{-2}$	$P_2''$ (Am)	$\theta_2$	$\mu_2$	$x_2$ (m) $\times 10^{-2}$	$F_2^C$ (N) $\times 10^{-3}$	$F_2^I$ (N) $\times 10^{-3}$
3.107	5.3	4.3	14.50	0.50	21.974	$29^0$	0.554	1.0	116.1	168.7

$$F^1_{Correction} = 24.2 \times 10^{-3} N. \quad F^2_{Correction} = 4.3 \times 10^{-3} N.$$

The correctness of our result entirely depends upon the value of the terrestrial magnetism. We have already performed the present experiment in a science exhibition [4].

## REFERENCES

1. Electricity and magnetism, A.S. Ramsey, Cambridge University Press, 2009.
2. Physics 2, NCERT Text Book for Class XII, 2011.
3. Physics 1, NCERT Text Book for Class XI, 2011.
4. The Science, Technology, Engineering & Math (STEM) Talent Search, AS & S Science & Sustainability Workshop Series, Kolkata, Organized jointly by Association of Science & Society, Harlem Children Society, New York, USA and Rockefeller University Chapter of Sigma Xi Scientific Research Society, New York, USA on 05.05.2012.

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