Flow of MHD Thermal Stagnation Point Flow of Micropolar Fluids due to Permeable Stretching Surface

Abstract:

This article examine the flow of micropolar fluids toward a porous stretching surface. The fluids is electrically conducting and the temperature is due to convection, radiation and joule's heating. A magnetic field in the normal direction is applied for the stagnation point flow. The governing model of the problem has been converted into ordinary differential form with the use of similarity functions. The final mathematical model is treated by coding in Mathematica. Several computation have been made for suitable ranges of the pertinent parameter that influence of flow pattern internal micromotion and thermal behaviour of a problem. The results have been presented in the form of plots for heat function of micromotion and speed of the flow.

Introduction:

Eringen [1,2]firstly formulated the theory of micropolar fluids and derived the constitutive laws for the fluids with microstructure. This theory provided a mathematical model for the non-Newtonian behavior which could be observed in certain liquids such as polymers, colloidal suspensions, animal blood, liquids crystals etc. A thorough review of the subject and application of micropolar fluid mechanics was provided by Ariman et al. [3,4] and Eringen [5]. Subhadra et al. [6] studied the heat transfer in the flow of a micropolar fluid past a curved surface with suction and injection using Van Dyke's singular perturbation technique. Muhammad Ashraf et al. [7] and Rashidi et al. [8] examined the steady, incompressible and laminar flow of micropolar fluids inside an infinite channel. The governing equations were reduced to non-linear ordinary differential equations by using similarity transformation. These equations were then solved using numerical procedures which included SOR method. Kelson and Farrell [9]analyzed self-similar boundary layer flow of a micropolar fluid in a porous channel where the flow was driven by uniform mass transfer through the channel walls. Ziabakhsh and Domairry [10], Joneidiet al. [11] also discussed the micropolar fluid in a porous channel with stationary walls by HAM or OHAM, respectively. Si et al. [12] firstly investigated the flow through a porous channel with expanding or contracting walls by HAM and analyzed the effects of the expansion ratio on the velocity and micropolar velocity.Kishore et al. [13] described the incompressible viscous hydromagnetic flow in a porous medium in the presence of radiation, variable heat and viscous dissipation and mass diffusion. Poornima et al. [14] studied the mathematical solution of a steady no cost convective flow of boundary-layer fluid flow of a radiating combined nanofluid to a non-linear boundary moving sheet in presence involving transverse magnet field. Sharma et al. [15] investigated the heat transfer due to exponentially shrinking sheet in the existence of the thermal radiation among mass suction of the boundary layer flow of a viscous fluid. Seddeek [16] analyzed the effects of radiation and variable viscosity on an MHD free convection flow past a semi-infinite flat plate with an aligned magnetic field. Hunegnew [17] studied the MHD boundary layer flow and heat transfer over a non-linearly stretching/shrinking sheet. Rajesh [18] investigated chemical reaction and radiation effects on the transient MHD free convection flow of dissipative fluid past an infinite vertical porous plate with ramped wall temperature. Khan and Sanjayan [19] reported an analytical solution of the viscoelastic boundary layer flow and heat transfer over an exponentially stretched sheet considering the viscous dissipation in the heat transfer equation.

Mathematical Model:

Consider micropolarfluid flow towards the stagnation point on a porous stretching surface**.** The fluid is incompressible and electrically conducting. The magnetic field of strength $H_a$is perpendicular to the surface that stretches or shrinks along x-axis. The flow is steady and two-dimensional. The horizontal component of velocity varies proportional to a specified distance x. The surface temperature is u_w and the free stream velocity u_∞ were assumed to vary proportional to the distance x from the stagnation point so that u_w =ax and u_∞ =bx. The temperature in the region exterior to the boundary layer isT_∞ . The induced magnetic field due to motion of the electrically conducting fluid and the pressure gradient are neglected. The tangential temperature is maintained at the prescribed constant value T_w . The body couple is absent. Spin vector is$\underline{\omega} = \omega(0,0,\omega_3)$

and Velocity vector is $\underline{V} = V(u,v)$

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The boundary layer Governing equations of the problems are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}) = a^2 x + (\mu + k)\frac{\partial^2 u}{\partial y^2} + k\frac{\partial \omega_3}{\partial y} - \frac{\sigma B_0^2}{\rho} u \tag{2}$$

$$\gamma(\frac{\partial^2 \omega_3}{\partial y^2}) - \kappa(\frac{\partial u}{\partial y} + 2\omega_3) = \rho j\left(u\frac{\partial \omega_3}{\partial x} + v\frac{\partial \omega_3}{\partial y} \right) \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p}(\frac{\partial u}{\partial y})^2 + \frac{16\alpha}{3\beta\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{\sigma B_0^2}{\rho C_p}u^2 \tag{4}$$

Where ρ is density of the liquid, μ is dynamic viscosity, B_0 is the strength of the applied magnetic field, α is the thermal diffusivity, σ is the electrical conductivity, c_p is the specific heat capacity at constant pressure, k and γ are additional viscosity coefficients for micropolar fluid and j is micro inertia ,

The boundary conditions are:

$$\omega_3(x,0) = 0,\ u(x,0) = bx,\quad v(x,0) = -v,\quad T(x,0) = T_w \tag{5}$$

$$\omega_3(x,\infty) = 0,\ u(x,\infty) = ax,\ T(x,\infty) = T_\infty$$

Where c is the proportionality constant of the velocity of the stretching sheet, a is the constant proportional to the free stream velocity for away from the sheet and T_∞ is the temperature of the ambient fluid.

Using similarity transformations:

The velocity components are described in terms of the stream function Ψ (x, y):

$$u = \frac{\partial \psi}{\partial y},\quad v = -\frac{\partial \psi}{\partial x}$$

$$\psi(x,y) = x\sqrt{c\upsilon}f(\eta),\ \eta = y\sqrt{\frac{c}{\upsilon}}$$

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$$u = xcf' \quad, \quad v = -\sqrt{\upsilon c}\, f \quad, \quad \omega_3 = \frac{c^{\frac{3}{2}}}{v^{\frac{1}{2}}} xL(\eta) \quad,$$

$$\theta\,(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$$

Equation of continuity (1) is identically satisfied.

Substituting the above appropriate relation in equations (2), (3) and (4) we get

$$(1+d_1) f''' + d_1 L' - H_a f' + \lambda^2 = f'^2 - ff'' \tag{6}$$

$$d_3 L'' + 2d_1 d_2 L - d_1 d_2 f'' = f'L - fL' \tag{7}$$

$$(4 + 3R_n)\theta'' + 3R_n \Pr(f\theta' + E_c f''^2 + H_a G_T f'^2) = 0 \tag{8}$$

The associated boundary conditions are:

$$f'(0) = 1,\; f(0) = -f_w,\; L(0) = 0,\; \theta(0) = 1, \tag{9}$$
$$f'(\infty) = \lambda,\; L(\infty) = 0,\; \theta(\infty) = 0,$$

Whereas $H_a = \dfrac{\sigma B_0^{\,2}}{\rho a}$ is the Magnetic parameter, $P_r = \dfrac{\upsilon}{\alpha}$ id the Prandtl number, $G_T = \dfrac{b^2 x^2}{C_p(T_w - T_\infty)}$ is the Grashof

number, $E_c = \dfrac{u_e^{\,2}}{C_p(T_w - T_\infty)}$ is the Eckert number, $R_n = \dfrac{\beta k}{4\alpha T_\infty^3}$ is Radiation parameter, $\lambda = \dfrac{a}{b}$ is the velocity ratio parameter and

$f_w = \dfrac{v}{\sqrt{b\upsilon}}$ is the suction/injection parameter.

The dimensional less material constants are

$$d_1 = \frac{k}{\mu},\, d_2 = \frac{\mu}{\rho\, j\, a},\, d_3 = \frac{\gamma}{\rho\, j\upsilon}$$

1. Results and Discussion:

The set of equations (6) to (9) is the highly non-linear and do not lend foe analytical solution. Numerical solution of the problem has been sought through Mathematica. In order to compute the effect of the physical parameters, namely $\lambda, H_a, G_r, R_n, E_c, P_r$ and the micropolar parameters d_1, d_2, d_3 on heat distribution and flow dynamics, computations, are made for sufficient ranges for these parameters when $\kappa = 0$ $(d_1 = 0)$ and $\Omega = 0$, the problem becomes same as the Newtonian fluid flow. The plots of the results has been presented to demonstrate the nature of the problem.

The velocity f' reduced in magnitude with increase in magnetic field strength as presented in fig 1. The result is accordance in physical situation because the Lorentz's force acts in opposite direction of the fluid. Fig 2 displays the velocity f' under the effect of d_1 the velocity increases with d_1.

Fig.3 demonstrate the velocity increase for suction but decreases for increase in injection. The effect of stretching parameter on the velocity is plotted in fig .4 when $\lambda < 1$. It is noticed that the velocity increases with λ.

The increase in microrotation parameters d_1 cause increase in microrotation as shown in fig 5. The microrotation increase with suction but decreases in injection near the boundary and reverse effect is observed away from the boundary as depicted in fig 6. Fig 7 demonstrate the effect of magnetic force in microrotation decreases near the boundary but increases in magnitude away from the boundary increase in H_a.

The heat function $\theta[\eta]$ decreases with increase in P_r and R_n as shown in fig 8 and fig 9. But the temperature distribution with increase in E_c and G_T as demonstrate respectively fig 10 and fig 11.
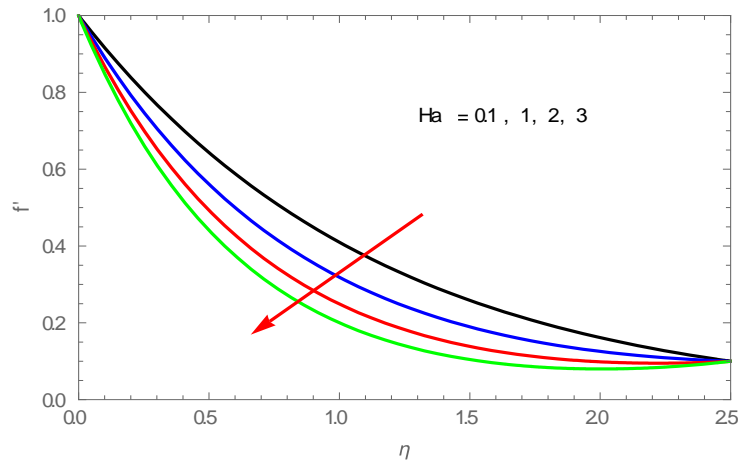

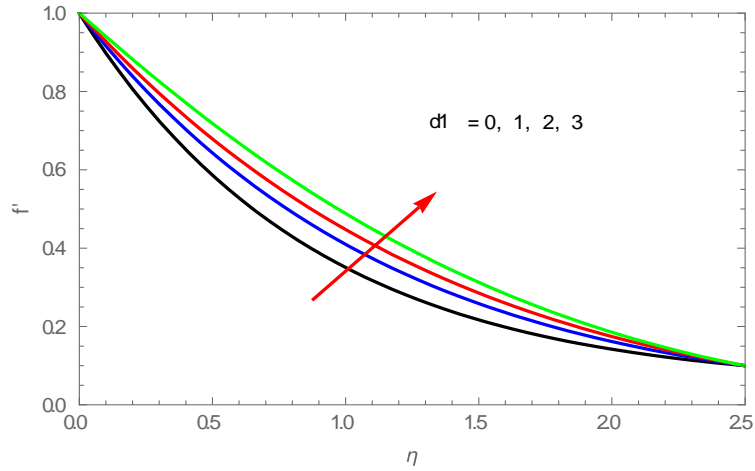**Fig.1: The plot for curves of f' under the effect of Hartmann number H_a**

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Fig.2: The plot for curves of f' under the effect of micropolar parameter d_1
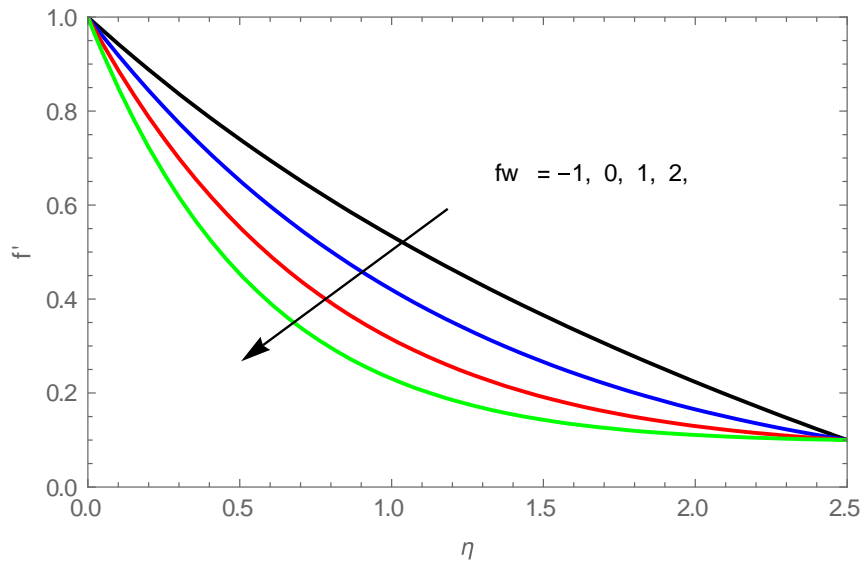

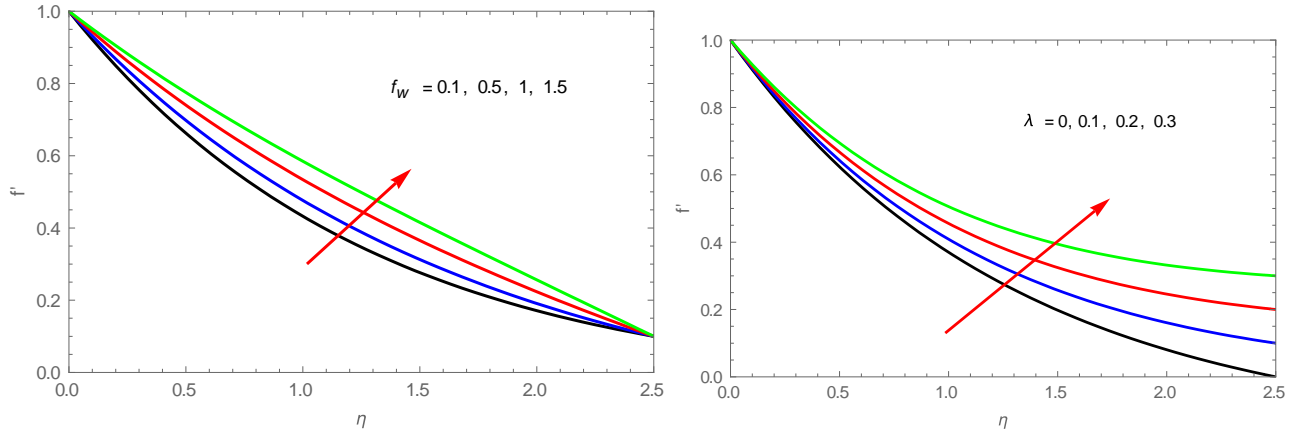**Fig.3: The plot for curves of f' under the effect of suction/ injection parameter A**

Fig.4: The plot for curves of f' **under the effect of** f_w .Fig.5: The plot for curves of f' under the effect of λ
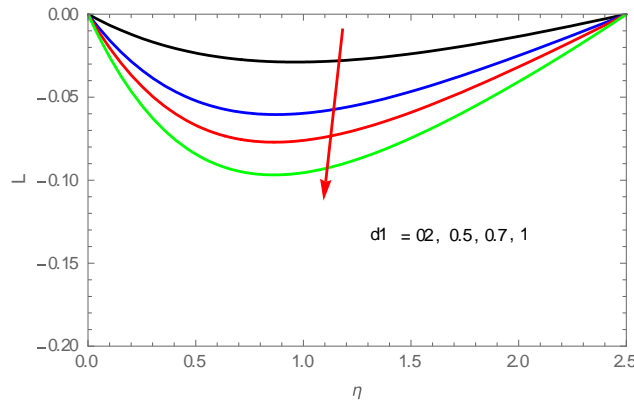

Fig.6: The plot for curves of microrotation L **under the effect of** d_1
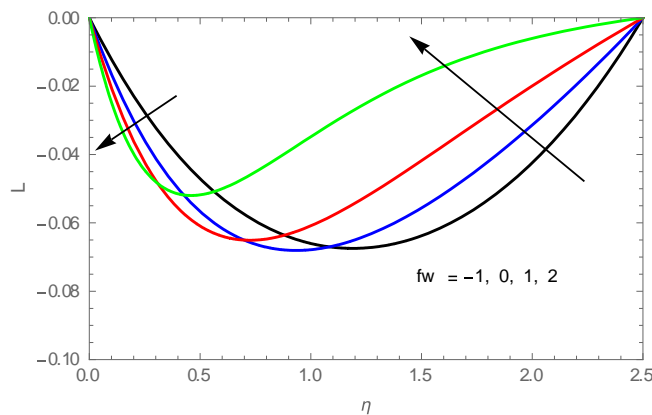

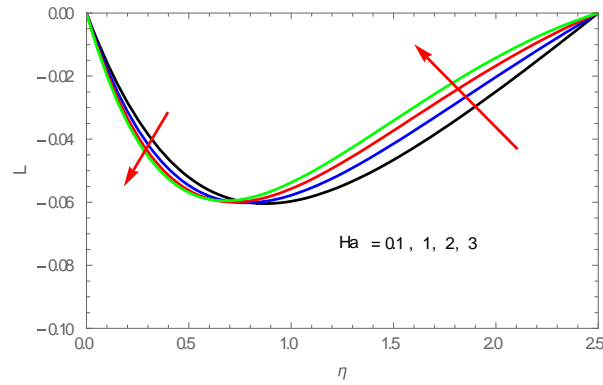**Fig.7: The plot for curves of L under the effect of suction/ injection parameter** *A*

Fig.8: The plot for curves of microrotation L under the effect of Hartmann number H_a
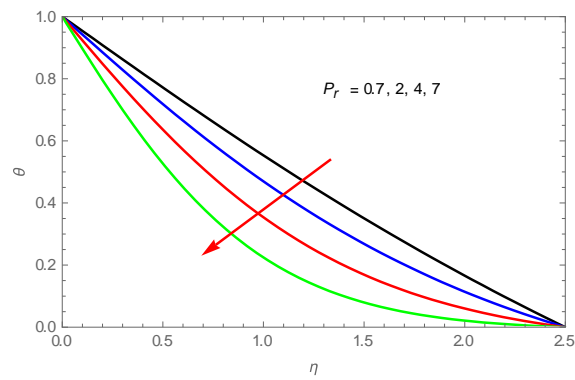

Fig.9: The plot for curves of θ under the effect of Prandtl number P_r
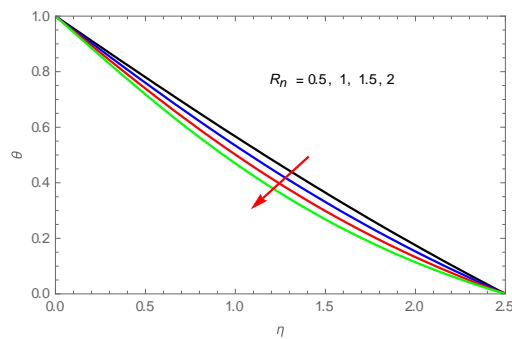

Fig.10: The plot for curves of θ under the effect of Radiation parameter R_n
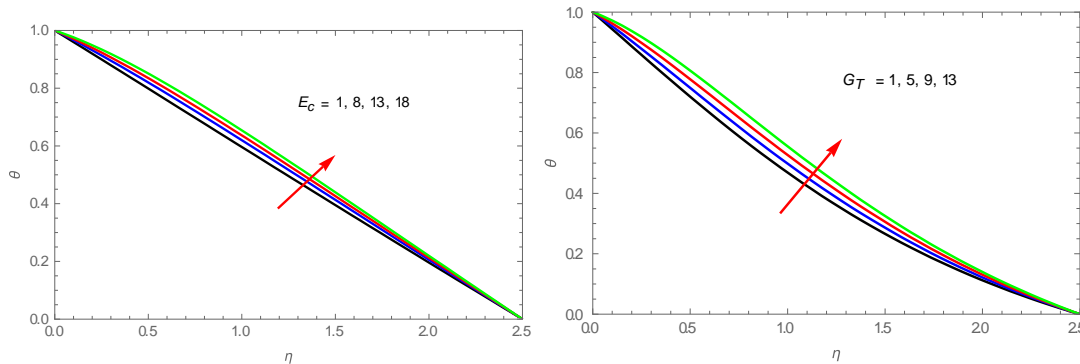
Fig.11: The plot for curves of θ under the effect of Eckert number E_cFig.12: The plot for curves of θ under the effect of G_T

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Author(s) & Affiliation

Hassan Waqas[1], Sajjad Hussain[2], Shamila Khalid[3]

[2]Punjab Higher Education Department, College Wing, Lahore, Pakistan.
[1,3]Department of Mathematics, Chenab College of Advance Studies, Faisal Abad, Pakistan.
[1,3]Presently: Department of Mathematics, Govt College University Faisal Abad (Layyah Campus), Pakistan.