THE EXPERIMENT
INTERNATIONAL JOURNAL OF SCIENCE AND TECHNOLOGY

ISSN-2319-2119

RESEARCH ARTICLE

V. B. Ghute et al, The *Experiment, 2014, Vol.24 (4)1683-1691*

NONPARAMETRIC CONTROL CHARTS FOR VARIABILITY USING RUNS RULES

ABSTRACT

In this paper, two Shewhart-type nonparametric control charts based on runs rules are developed for monitoring the changes in the process variability. The charts are based on two nonparametric tests for equality of variances. The performance of the proposed control charts is evaluated for the normal, light tailed and heavy tailed distributions through average run length using simulations.

Keywords Average run length, runs rules, process variability, nonparametric tests.

1. INTRODUCTION

Control charts are statistical process control tools that are widely used to detect unfavorable changes in a process with respect to some quality characteristics. The changes may occur in average as well as in the variability of the relevant quality characteristics. It is, therefore, essential to control both of them using by two control charts, one chart for process average and another chart for process variability. Most of the control charts that are developed for monitoring process average and process variability are designed and evaluated under the assumption that the underlying distribution of the quality characteristic is normal. In real applications, there are many situations in which the process data come from a non-normal distribution which need to be monitored by appropriate control charts. To monitor such type of data, development of control charts that do not depend on a particular distributional assumption is desirable. Nonparametric control charts can serve this purpose. The main advantage of a nonparametric control chart is that it does not assume any probability distribution for the characteristic of interest. A formal definition of nonparametric or distribution-free control chart is given in terms of its in-control run length distribution. The number of samples that needs to be collected before the first out-of-control signal given by a chart is a random variable called the run-length; the probability distribution of the run-length is referred to as the run-length distribution. If the in-control run length distribution is same for every continuous distribution then the chart is called distribution-free or nonparametric [1].

In literature, several nonparametric control charts are proposed for monitoring location of a univariate process. Some of these are based on sign and/or rank statistics by assuming a known in-control target value for process location. An extensive overview of the literature on univariate nonparametric control charts is presented in [2]. A distribution-free Shewhart control chart for monitoring process center based on the signed-ranks of grouped observations is developed in [3]. Shewhart, CUSUM and EWMA control charts based on signed-rank-like statistics of grouped data for monitoring a process center when in-control target center was not specified is proposed in [4] and studied the robustness of the charts against outliers. A nonparametric charts based on runs rules of Wilcoxon's signed-rank statistic is developed in [5]. A class of nonparametric Shewhart-type charts based on runs rules of sign statistic is developed in [6].

However, there are few nonparametric charts in literature to monitor change in process variability. Using non-parametric tests for the equality of two variances for use as control statistics in nonparametric control charts for variability is suggested in [7]. Control charts using tests statistics for comparing two variances would require obtaining an initial sample (of size m) when the process is considered to be in-control. Then at each sample time i, a sample of size n is obtained from the process, and the pooled sample of size (m + n) is obtained. The observations in the pooled sample then are ranked from smallest to largest, and some statistic based on the ranks of the observations is calculated. A nonparametric control chart for monitoring process variability based on Conver's squared rank test for variance is proposed in [8]. Two nonparametric control charts for monitoring process variability based on two nonparametric tests are developed in [9]. A nonparametric control chart for dispersion based on the rank sum statistic is proposed in [10].

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When the process distribution is normal, Shewhart R and S charts are appropriate control chart for monitoring the process variability. If underlying process distribution is non-normal, then the need of development of nonparametric control chart based on appropriate nonparametric test arises. In this paper, we introduce two Shewhart-type nonparametric control charts using runs rules for monitoring process variability for the case that the location parameter is under control. The proposed nonparametric control charts are based on two sample nonparametric tests proposed in [11] and [12]. These are most powerful test statistics for detecting scale shifts. The performance of the proposed charts is assessed for both the in-control state and out-of-control state under different underlying distributions.

2. MATERIAL AND METHOD

2.1. Nonparametric control chart based on Sukhatme test

A nonparametric test for two independent samples dispersion problem is proposed in [11]. Suppose we want to compare two independent random samples $X=(X_1,X_2,...,X_m)$ and $Y=(Y_1,Y_2,...,Y_n)$ which are drawn from absolute continuous distributions and differ only in the scale parameters. Let σ_X and σ_Y be the arbitrary measures of dispersion of X and Y respectively then problem of testing of hypothesis is $H_0:\sigma_X=\sigma_Y$ against $H_1:\sigma_X\neq\sigma_Y$. The Sukhamte test statistic for testing null hypothesis is defined as,

$$T=\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}D(X_i,Y_j)\ , \tag{1}$$

$$\begin{aligned} \text{where}\quad D(X,Y) &= 1 \quad \text{if either } 0<X<Y \text{ or } Y<X<0 \\ &= 0 \quad \text{otherwise} \end{aligned}$$

We reject hypothesis if T is too large or too small. The mean and variance of the statistic T is given by, $E(T)=\frac{1}{4}$ and

$$Var\ (T)=\frac{(m+n+7)}{48\,m\,n}$$

For a large sample,

$$Z=\frac{T-E(T)}{\sqrt{Vat\,(T)}} \tag{2}$$

has a standard normal distribution and the test is performed on the basis of tabulated values of the standard normal distribution.

We consider Z as the control chart statistic for the nonparametric control chart for monitoring process variability and the chart is referred as NP-S chart. We consider $X=(X_1,X_2,...,X_m)$, as reference sample of size m from an in-control process and that $Y=(Y_1,Y_2,...,Y_n)$ be an arbitrary test sample of size n. The sample statistics Z computed from independent observations from the process are plotted against an upper control limit UCL = 3 and LCL = -3. The process is considered out-of-control when a plotted point lies above UCL or below LCL.

2.2. Nonparametric control chart based on Mood test

A nonparametric test for equality of variances is developed in [12]. Suppose we have two independent random samples

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$X = (X_1, X_2, ..., X_m)$ and $Y = (Y_1, Y_2, ..., Y_n)$. We wish to test $H_0 : \sigma_X = \sigma_Y$ against $H_1 : \sigma_X \neq \sigma_Y$. Let $R_1 < R_2 < .... < R_m$ be the combined samples ranks of the X-values in increasing order of magnitude. The Mood test statistic for testing null hypothesis is defined as,

$$M = \sum_{i=1}^{m} \left(R_i - \frac{N+1}{2} \right)^2, \quad \text{where } N = m + n \tag{3}$$

The mean and variance of the statistic M is given as

$$E(M) = \frac{m(N^2 - 1)}{12} \quad \text{and} \quad Var(M) = \frac{mn(N+1)(N^2 - 4)}{180}$$

For N greater than or equal to 30, we may consider the normalized random variable W

$$W = \frac{M - E(M)}{\sqrt{Var(M)}} \tag{4}$$

and perform the test on the basis of tabulated values of the standard normal distribution.

For N less than 30, it is not advisable to use directly the normal approximation. In that case, Laubscher recommended the use of a correction for continuity yielding the following test statistic:

$$W = \frac{M - E(M)}{\sqrt{Var(M) + \dfrac{1}{2\,Var(M)}}} \tag{5}$$

and perform the test again based on the tabulated values of the standard normal distribution.

We consider W as the control chart statistic for the nonparametric control chart for monitoring process variability and the chart is referred as NP-M chart. We consider $X = (X_1, X_2, ..., X_m)$, as reference sample of size m from an in-control process and that $Y = (Y_1, Y_2, ..., Y_n)$ be an arbitrary test sample of size n. The sample statistics W computed from independent observations from the process are plotted against an upper control limit UCL = 3 and LCL = -3. The process is considered out-of-control when a plotted point lies above UCL or below LCL.

2.3. Nonparametric charts with runs rules

In this section, we study the performance of the proposed NP-S and NP-M control charts using runs rules. A Shewhart-type nonparametric control chart based on Wilcoxon's signed-rank statistic and the following run types rules is developed in [5]. Note that we would usually use only one of these rules at a time. Each rule would have different control limits.

The process is declared to be out of control when:
a) single point plots outside the control limit (one-of-one rule); or
b) k consecutive points plot outside the control limit (k-of-k rule); or
c) k of the last w points plot outside the control limits (k-of-w rule).

In order to study the performance of NP-S and NP-M charts we consider rule (a) and (b) of [5]. The ARL performance of NP-S and NP-

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M charts with runs rules is investigated for k = 2 using a simulation study. The one-of-one chart signals an out-of-control status if a point falls either above upper control limit or below a lower control limit. The one-of-one chart is Shewhart-type control chart. The two-of-two chart signals an out-of-control status if two consecutive points fall either above an upper control limit or below lower control limit.

3. RESULTS AND DISCUSSION

To examine the ability of proposed NP-S and NP-M charts to detect variability shift in a process, we consider underlying process distributions as normal, double exponential and uniform with mean zero and variance one. The uniform distribution is considered as process distribution to see the effect of a light tailed distribution and double exponential distribution is considered to see the effect of heavy tailed distribution on the performance of proposed nonparametric control charts. Consider a process where quality characteristic of interest X is distributed with mean μ and standard deviation σ. Let μ_0 and σ_0 be the in-control values of μ and σ respectively. When a shift in process standard deviation occurs, we have change from the in-control value σ_0 to the out-of-control value $\sigma_1 = \delta\sigma_0$ $(0 < \delta \neq 1)$. Therefore, when control chart for variability is employed, the process shifts are measured through $\delta = \dfrac{\sigma_1}{\sigma_0}$.

When $\delta = 1$, the process is considered to be in-control . For $\delta > 1$ an increase in σ occurs and for $\delta < 1$, decrease in σ occurs. In the present study only increase in process variability is considered. Computer programs written in C language are used to study the performance of the proposed control charts. The in-control and out-of-control ARL values of the proposed control charts are computed using 10000 simulations for sample size of n = 10, 15, 20 and 25.

Table 1 to Table 4 provide the ARL values of the proposed NP-S chart with one-of-one and two-of-two rules when the underlying process data actually follows normal, double exponential and uniform distributions with sample sizes n =10, 15, 20 and 25 respectively. Table 5 to Table 8 provide the ARL values of the proposed NP-M chart with one-of-one and two-of-two rules when the underlying process data actually follows normal, double exponential and uniform distributions with sample sizes n =10, 15, 20 and 25 respectively.

Examinations of Table 1 to Table 8 lead to the following findings:

- In-control ARL values of the proposed NP-S and NP-M control charts with one-of-one rule are almost identical under different process distributions.
- In-control ARL values of the proposed NP-S and NP-M control charts with two-of-two rule are almost identical under different process distributions.
- Out-of-control ARL values of NP-S and NP-M charts with two-of-two rule are smaller than that of one-of-one rule for each process distribution. Therefore, the charts with two-of-two rule detect shift in process variability earlier than one-of-one rule.
- Out-of-control ARL values of NP-M chart with one-of-one and two-of-two rules are smaller than the corresponding ARL values of the NP-S chart. NP-M chart is more efficient than NP-S chart for normal, light tailed uniform and heavy tailed double exponential distributions.
- For normally distributed data, both NP-S and NP-M charts performs better than double exponential data.
- For uniformly distributed data, both NP-S and NP-M charts perform better than normally and doubly exponential data.

- Performance of the proposed charts also depends on sample size. For increase in sample size indicates the improvement in the ability to detect shift in process variability.

4. CONCLUSION

In this paper, two nonparametric control charts based on runs rules are developed for monitoring process variability. The performance of the proposed control charts is studied by simulation under normal, light tailed and heavy tailed distributions. It indicates that performance of the proposed charts is improved when runs rules are used. Our simulation study indicates that the NP-M control chart is more efficient than NP-S control chart for detecting shifts in process variability for different process distributions. Both NP-M and NP-S control charts perform better when underlying process distribution is light tailed.

ACKNOWLEDGEMENT

The authors would like to thank the Editor and the referees for their valuable comments and helpful suggestions for the improvement of this paper. The first author would like to acknowledge the support by University Grants Commission, New Delhi for award of Teacher Fellowship under Faculty Improvement Program during the XIIth plan.

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Shift δ	One-of-one rule			Two-of-two rule		
	Normal	Double Exponential	Uniform	Normal	Double Exponential	Uniform
1.0	481.53	474.32	478.78	520.12	528.11	525.76
1.2	217.34	260.98	162.64	232.38	292.22	143.12
1.4	156.23	155.24	72.45	92.91	144.58	45.03
1.6	70.12	100.61	40.37	44.66	77.28	20.55
1.8	46.28	71.44	26.78	25.43	45.48	12.52
2.0	33.24	57.82	19.39	16.37	30.29	8.68
2.2	25.03	41.67	14.68	11.77	21.54	6.81
2.4	20.01	33.49	11.78	8.90	16.17	5.51
2.6	16.31	27.55	9.87	7.33	12.87	4.70
2.8	13.67	23.50	8.43	6.14	10.44	4.26
3.0	11.96	20.03	7.45	5.34	8.88	3.87
4.0	6.90	11.40	4.65	3.58	5.18	2.96
5.0	4.92	7.78	3.67	2.96	3.84	2.62

Table-1: ARL values of NP-S chart when n = 10

Shift δ	One-of-one rule			Two-of-two rule		
	Normal	Double Exponential	Uniform	Normal	Double Exponential	Uniform
1.0	448.66	451.61	454.26	554.12	556.26	557.30
1.2	168.67	211.34	105.46	203.75	227.08	106.57
1.4	69.17	104.24	35.19	61.69	106.37	25.82
1.6	34.42	56.67	16.38	24.89	47.89	11.02
1.8	19.53	35.16	9.59	13.47	26.13	6.65
2.0	12.66	23.62	6.36	8.37	16.32	4.78
2.2	8.86	16.76	4.69	6.11	11.22	3.89
2.4	6.68	12.53	3.78	4.84	8.50	3.26
2.6	5.22	9.95	3.15	4.08	6.71	2.92
2.8	4.48	8.00	2.70	3.49	5.56	2.75
3.0	3.69	6.59	2.43	3.19	4.79	2.59
4.0	2.17	3.49	1.66	2.44	3.07	2.24
5.0	1.68	2.41	1.40	2.24	2.54	2.12

Table-2: ARL values of NP-S chart when n = 15

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Shift δ	One-of-one rule			Two-of-two rule		
	Normal	Double Exponential	Uniform	Normal	Double Exponential	Uniform
1.0	543.07	545.49	537.98	503.71	507.03	506.82
1.2	161.15	220.82	93.30	153.95	224.31	72.02
1.4	54.46	89.86	24.05	38.86	72.17	15.34
1.6	22.95	43.65	10.06	14.72	29.94	6.75
1.8	12.48	24.46	5.72	7.95	15.61	4.28
2.0	7.60	14.95	3.81	5.29	9.72	3.30
2.2	5.13	10.38	2.80	3.99	6.77	2.81
2.4	3.86	7.49	2.27	3.32	5.19	2.50
2.6	3.06	5.77	1.93	2.88	4.26	2.36
2.8	2.56	4.59	1.73	2.64	3.65	2.25
3.0	2.19	3.82	1.56	2.44	3.24	2.18
4.0	1.43	2.09	1.22	2.12	2.39	2.05
5.0	1.22	1.55	1.11	2.05	2.16	2.03

Table-3: ARL values of NP-S chart when n = 20

Shift δ	One-of-one rule			Two-of-two rule		
	Normal	Double Exponential	Uniform	Normal	Double Exponential	Uniform
1.0	513.33	503.57	523.46	516.60	514.31	510.23
1.2	132.85	188.11	68.51	119.92	186.98	50.86
1.4	38.12	66.96	15.02	26.53	51.41	10.23
1.6	23.39	29.72	6.21	9.88	20.02	4.75
1.8	12.43	15.55	3.48	5.46	10.31	3.26
2.0	7.42	9.32	2.45	3.84	6.62	2.66
2.2	5.23	6.47	1.88	3.06	4.79	2.35
2.4	3.91	4.65	1.58	2.63	3.84	2.21
2.6	3.06	3.56	1.42	2.41	3.22	2.13
2.8	2.55	2.92	1.31	2.27	2.87	2.09
3.0	2.16	2.45	1.23	2.19	2.61	2.06
4.0	1.44	1.48	1.07	2.04	2.14	2.01
5.0	1.21	1.20	1.02	2.01	2.04	2.00

Table-4: ARL values of NP-S chart when n = 25

Shift δ	One-of-one rule			Two-of-two rule		
	Normal	Double Exponential	Uniform	Normal	Double Exponential	Uniform
1.0	328.93	332.94	335.48	251.17	253.42	255.79
1.2	135.24	174.71	85.03	105.55	136.49	54.28
1.4	62.06	92.96	30.26	42.36	67.19	17.30
1.6	33.09	54.86	14.83	20.37	37.02	8.98
1.8	20.13	36.10	9.16	12.53	23.24	6.07
2.0	13.66	24.69	6.43	8.57	15.74	4.58
2.2	9.91	18.15	5.01	6.46	11.85	3.91
2.4	6.82	12.63	3.71	5.28	9.23	3.47
2.6	6.13	11.67	3.44	4.55	7.62	3.20
2.8	5.13	9.51	2.99	4.02	6.41	3.01
3.0	4.39	7.96	2.69	3.65	5.60	2.87
4.0	2.71	4.45	1.95	2.84	3.74	2.49
5.0	2.10	3.16	1.67	2.54	3.04	2.38

Table-5: ARL values of NP-M chart when n = 10

Shift δ	One-of-one rule			Two-of-two rule		
	Normal	Double Exponential	Uniform	Normal	Double Exponential	Uniform
1.0	420.16	415.82	411.57	388.93	389.19	389.00
1.2	137.21	184.70	66.75	118.52	171.04	45.97
1.4	46.02	78.83	17.62	33.16	63.47	11.07
1.6	20.50	39.69	7.49	14.13	28.34	5.47
1.8	11.25	23.02	4.53	7.77	15.77	3.76
2.0	7.10	14.85	3.19	5.28	10.12	3.06
2.2	5.00	10.45	2.44	4.07	7.37	2.66
2.4	3.59	6.75	1.91	3.37	5.69	2.46
2.6	3.05	5.95	1.80	2.97	4.67	2.34
2.8	2.59	4.83	1.64	2.71	4.05	2.25
3.0	2.23	4.05	1.53	2.53	3.54	2.20
4.0	1.49	2.26	1.23	2.19	2.58	2.08
5.0	1.28	1.71	1.16	2.10	2.29	2.05

Table-6: ARL values of NP-M chart when n = 15

Shift δ	One-of-one rule			Two-of-two rule		
	Normal	Double Exponential	Uniform	Normal	Double Exponential	Uniform
1.0	418.01	414.41	416.31	438.46	435.84	441.62

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1.2	109.29	166.97	45.11	99.98	160.75	32.42
1.4	31.19	58.02	10.23	22.69	48.39	7.43
1.6	12.73	26.67	4.42	9.13	19.82	3.92
1.8	6.61	14.51	2.65	3.75	10.56	2.85
2.0	4.09	8.92	1.95	3.70	6.73	2.44
2.2	2.93	6.11	1.58	3.0	4.94	2.25
2.4	2.28	4.50	1.42	2.59	3.95	2.15
2.6	1.87	3.46	1.28	2.37	3.34	2.09
2.8	1.66	2.87	1.20	2.25	2.94	2.06
3.0	1.47	2.44	1.16	2.18	2.71	2.05
4.0	1.15	1.51	1.05	2.04	2.18	2.01
5.0	1.07	1.24	1.03	2.01	2.07	2.01

Table-7: ARL values of NP-M chart when n = 20

Shift δ	One-of-one rule			Two-of-two rule		
	Normal	Double Exponential	Uniform	Normal	Double Exponential	Uniform
1.0	418.44	424.64	426.27	468.41	463.56	460.16
1.2	90.68	139.41	33.15	82.20	141.43	23.47
1.4	22.46	44.64	6.83	16.71	36.17	5.43
1.6	8.39	19.05	2.96	6.70	14.28	3.05
1.8	4.42	9.86	1.88	3.92	7.68	2.41
2.0	2.83	5.81	1.46	2.95	5.05	2.18
2.2	2.05	4.10	1.27	2.50	3.74	2.09
2.4	1.65	3.05	1.16	2.28	3.08	2.05
2.6	1.44	2.41	1.11	2.15	2.71	2.03
2.8	1.29	2.02	1.07	2.10	2.48	2.02
3.0	1.20	1.76	1.05	2.06	2.33	2.01
4.0	1.04	1.21	1.01	2.01	2.06	2.00
5.0	1.02	1.08	1.00	2.00	2.02	2.00

Table-8: ARL values of NP-M chart when n = 25

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