

EFFECT OF LOADING ON PLATE ANALYSIS

1. INTRODUCTION

Plates with cutouts in aerospace, civil, mechanical and marine structures are inevitable mainly for practical and design considerations. In many circumstances these structures are found to be exposed to in-plane loading. The applied load is seldom uniform and the boundary conditions may be completely arbitrary in practice. The buckling and vibration characteristics of stiffened plates with cutout with cutouts pose a tremendous challenge and must be properly understood in the design of such structures. The buckling and vibration characteristics of stiffened plates subjected to in-plane partial edge loadings are studied using finite element method. Buckling loads and vibration frequencies are determined for different plate aspect ratios, edge conditions.

Vibration and buckling calculations for rectangular plates subjected to non-uniform in-plane stress distribution were studied by different investigators [1-3]. The effects of stiffener location on vibration and buckling characteristics have been discussed. Yamiki [4] analyzed the plate subjected to locally distributed in-plane loading over a finite length of the edge at the center of two opposite edges. Baker and Pavolic [5] analyzed the stability of rectangular plates subjected to a pair of patch loading at the center of two opposite edges by using Ritz method.

2. Governing equation

The equation of equilibrium for the stiffened plate subjected to in-plane loads can be written as:

$$[M] \{\ddot{q}\} + [[K_b] - P[K_G]] \{q\} = 0 \quad (1)$$

$$[[K_b] - P[K_G]] \{q\} = 0 \quad (2)$$

$$[[K_b] - P_{cr}[K_G] - \omega^2 [M]] \{q\} = 0 \quad (3)$$

3. Finite element formulation

The formulation is based on Mindlin's plate theory, which will allow for the incorporation of shear deformation.

The nine noded isoparametric quadratic element with five degree of freedom (u , v , w , θ_x , and θ_y) per node is employed in the present analysis. The coordinates at a point within the element are approximated in terms of its nodal co-ordinates as follows

$$x = \sum_{r=1}^9 N_r x_r \quad \text{and} \quad y = \sum_{r=1}^9 N_r y_r, \quad (4)$$

The geometric stiffness matrix is essentially a function of the in-plane stress distribution in the element due to applied edge loading. Since the stress field is non-uniform, for a given edge loading and boundary conditions, the static equation, i.e. $[K] \{\delta\} = \{F\}$ is solved to get these stresses. The geometric stiffness matrix is now constructed with the known in-plane stresses. The computer program developed accepts two sets of boundary conditions, the first for the static analysis and second for the buckling analysis. In the present case, a three-point integration scheme is adopted for the evaluation of all the matrices except the portion of the stiffness matrix related to shear strain components.

5. Results and Discussion

The problem considered here consists of a rectangular plate ($a \times b$) with longitudinal stiffeners having a central rectangular cutout of size ($g \times d$) subjected to harmonic in-plane partial edge loading at the plate boundary as shown in figure 1.

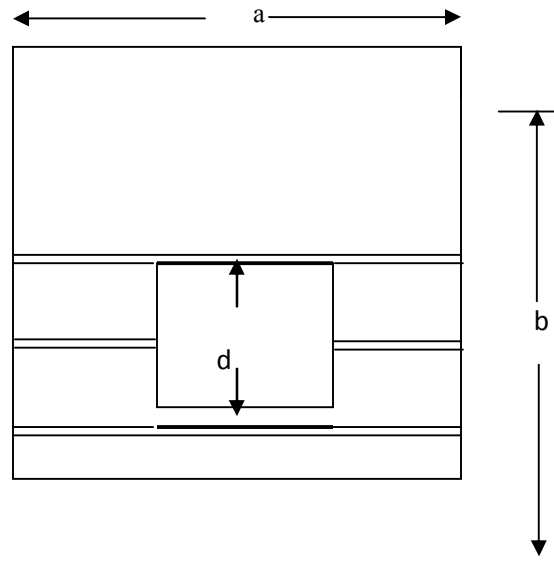


Figure 1 . Stiffened plate with cutout under point edge loading from both ends

To check the validity of the present finite element formulation and the corresponding programmes, two previously reported experimental and theoretical examples are analyzed by the finite element method for comparison. The figures for the examples taken here may be considered as shown in figure 2. The example is a single ribbed plate simply supported on all edges. A rectangular plate with one central stiffener is considered. The geometry and the material properties are as follows: $a = 600$ mm, $b = 410$ mm, $t = 6.33$ mm, $b_s = 12.7$ mm, $E = 211$ GPa, $\nu = 0.3$, $\rho = 7830$ Kg/mm³. Table 1 compares the present results and those presented by Harik and Guo [7], Aksu [8], Zeng and Bert [9] by differential quadrature analysis and Mukherjee and Mukhopadhyay [6]. The agreement is excellent.

Mode	Harik and Guo [7]	Aksu [8]	Zeng and Bert [9]	Ref [6]	Present
1	253.59	254.94	252.16	257.05	254.22
2	282.02	269.46	275.44	272.10	272.69
3	513.50	511.64	522.99	524.70	515.26

Table 1. Frequencies (Hz) for single ribbed plate with all edges simply supported

The case of partial edge loading at center is considered in this section. Numerical results for buckling, vibration analysis are presented for stiffened plate with cutout subjected to partial edge load at the centre. The influence of cutout size, aspect ratios, end conditions, stiffener parameters, and load width is studied in this section. The data used for the analysis are as follows: $b = 600$ mm, $b_s = 3.31$ mm, $d_s = 20.25$ mm, $\nu = 0.34$, $E = 6.87 \times 10^4$ N/mm², $\rho = 2.78 \times 10^{-6}$ Kg/mm³, $t = 1$ mm.

Numerical results for buckling load parameters are presented for stiffened square plates having one central stiffener subjected to in-plane partial edge load at the centre for simply supported and clamped boundary condition in three higher modes for the cases:

(i) With various central square cutout sizes ($g/a = 0.2, 0.4, 0.6, 0.8$) and load width ($c/b = 0.4$) in figure 3

It is observed from figure 3 that the buckling resistance decreases and the rate of decrease is noticed appreciably as the cutout size (g/a) increases up to 0.4. From cutout size (g/a) of 0.4 to 0.6, the rate of decrease is less. For cutout size more than 0.6, the variation is not significant for all modes and boundary conditions. It is also observed that the rate of decrease of buckling load parameter for simply supported case in first mode is less as compared to the higher modes. On the other hand, the rate of decrease of buckling load parameter for clamped case in all observed modes is almost uniform.

5. CONCLUSION

The stability resistance increases with increase of restraint at the edges for all types of loading, stiffener parameters and plate aspect ratios. The stability resistance increases with increase of number of stiffeners. Natural frequencies of stiffened plates always decrease with the increase of the in-plane compressive load. The fundamental frequency becomes zero at the respective values of the buckling load.

For partial edge load at one end, the vibration frequencies decrease with the increase of load width. The buckling load parameter of stiffened plate with cutout reduces with increase in the size of the hole. The frequency parameter values increase with the increase of cutout size. The rate of increase of frequencies also increases with increase in the size of the hole. The effect of cutout is to increase the vibration frequency of stiffened plate in almost all the cases.

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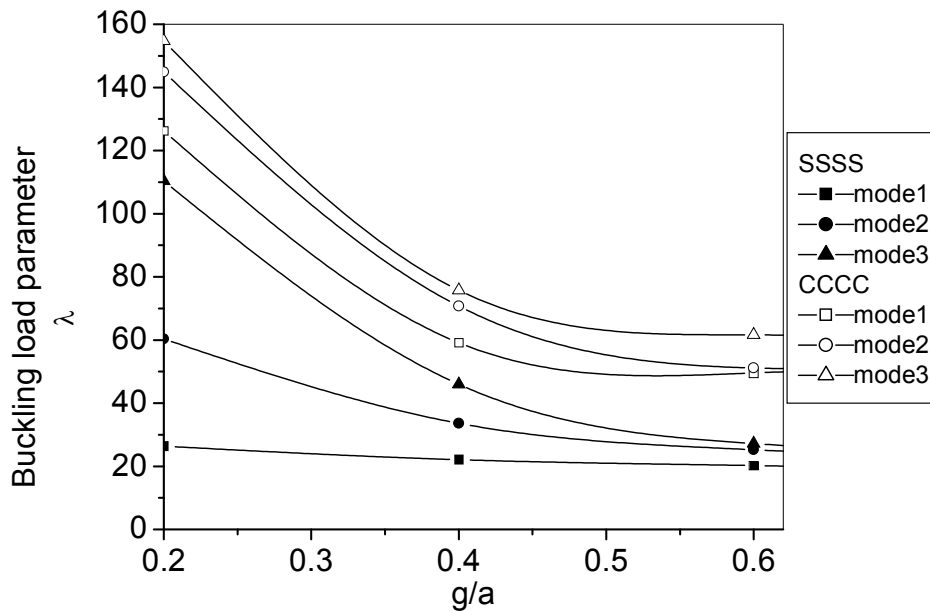


Figure 3. Variation of buckling load parameter with cutout size (g/a) of stiffened square plate having one central stiffener subjected to partial load at the centre. $c/b = 0.4$. The edges are SSSS and CCCC.

NOTATIONS

a	Plate dimension in longitudinal direction
b	Plate dimension in the transverse direction
t	Plate thickness
E, G	Young's and shear moduli for the plate material
ν	Poisson's ratio
b_s, d_s	web thickness and depth of a x-stiffener
ξ, η	Non –dimensional element coordinate
A_s	Cross sectional area of the stiffener
I_s	Moment of inertia of the stiffener cross-section about reference axis
$\{q\}_r$	Vector of nodal displacement a r^{th} node
$[D_p]$	Rigidity matrix of plate
$[D_s]$	Rigidity matrix of stiffener
$[K_e]$	Elastic stiffness matrix of plate
$[K_s]$	Elastic stiffness matrix of stiffener

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