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RESEARCH ARTICLE

M.A.Gopalan *et al*, The *Experiment*, *2014.*, *Vol. 18(1), 1218-1223*

On the Cubic Equation with Six Unknowns

$$x^3 + y^3 + z^3 + w^3 = 2(P^3 + Q^3)$$

ABSTRACT

The Cubic Equation $x^3 + y^3 + z^3 + w^3 = 2(P^3 + Q^3)$ is analyzed for its patterns of non – zero integral solutions. Four patterns of solutions are illustrated. A few properties among the solutions are presented.

KEY WORDS: Cubic Equation with Six Unknowns, Integral solutions**,** MSC2000 Subject Classification: 11D25

INTRODUCTION

Integral solutions for the homogeneous (or) non homogeneous Diophantine cubic equation is an interesting concept as it can be seen from [1, 2, 3]. In [4-9] a few special cases of cubic Diophantine equation with 4 unknowns are studied. In [10, 11], cubic equations with 5 unknowns are studied for their integral solutions. In this communication we present the integral solutions of an interesting cubic equation with 6 unknowns $x^3 + y^3 + z^3 + w^3 = 2(P^3 + Q^3)$.

NOTATIONS USED

- $t_{m,n}$ - Polygonal number of rank n with size m.
- p_n^m - Pyramidal number of rank n with size m.
- so_n - Stella octangular number of rank n
- s_n - Star number of rank n
- pr_n - Pronic number of rank n
- pt_n - Pentatope number of rank n

METHOD OF ANALYSIS

The Cubic Diophantine equation with six unknowns to be solved is given by

$$x^3 + y^3 + z^3 + w^3 = 2(P^3 + Q^3) \quad (1)$$

The substitution of the linear transformations

$$x = u + v \;,\; y = u - v \;\; z = u + s,\; w = u - s,\; P = u + r \text{ and } Q = u - r \quad (2)$$

in (1) leads to

$$v^2 + s^2 = 2r^2 \quad (3)$$

(3) is solved through different approaches and the different patterns of solutions of (1) obtained are presented below

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Pattern -I

Take $r = a^2 + b^2$ (4)

Write 2 as $2 = (1+i)(1-i)$ (5)

Substituting (4) & (5) in (3) and using the method of factorization, define

$$v + is = (1+i)(a+ib)^2 \qquad (6)$$

Equating the real and imaginary parts of (6), we get

$$\begin{aligned} v(a,b) &= a^2 - b^2 - 2ab \\ s(a,b) &= a^2 - b^2 + 2ab \end{aligned} \qquad (7)$$

Substituting (7) in (2), the integral solutions of (1) are given by

$$\begin{aligned} x &= u + a^2 - b^2 - 2ab \\ y &= u - a^2 + b^2 + 2ab \\ z &= u + a^2 - b^2 + 2ab \\ w &= u - a^2 + b^2 - 2ab \\ P &= u + a^2 + b^2 \\ Q &= u - a^2 - b^2 \end{aligned} \qquad (8)$$

Properties:

- $y(u,a,a(a+1)) + z(u,a,a(a+1)) - P(u,a,a(a+1)) - Q(u,a,a(a+1)) = 8p_a^5$
- $P(u(2u^2-1),a,b) + Q(u(2u^2-1),a,b) = 2so_u$
- $x(u,a,b) + y(u,a,b) = z(u,a,b) + w(u,a,b)$
- $x(u(u+1),a,b) + y(u(u+1),a,b) = 2pr_u$
- $z(u,a,a) - x(u,a,a) = y(u,a,a) - w(u,a,a)$ is a square number
- The triple $z(u,a,b) - x(u,a,b),\ \ x(u,a,b) + z(u,a,b) - 2u,\ \ 2P(u,a,b) - 2u$

  forms a Pythagorean triple

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Pattern -II

In (5), write 2 as

$$2 = \frac{(7+i)(7-i)}{25} \quad (8)$$

Following the procedure presented in pattern-I, the solution of (3) are given by

$$v = \frac{1}{5}(7a^2 - 7b^2 - 2ab)$$
$$s = \frac{1}{5}(a^2 - b^2 + 14ab) \quad (9)$$

Replacing *a* by *5A* and *b* by *5B* in (9), and using (2), we have

$$x(u,A,B) = u + 35A^2 - 35B^2 - 10AB$$
$$y(u,A,B) = u - 35A^2 + 35B^2 + 10AB$$
$$z(u,A,B) = u + 5A^2 - 5B^2 + 70AB$$
$$w(u,A,B) = u - 5A^2 + 5B^2 - 70AB$$
$$P(u,A,B) = u + 25A^2 + 25B^2$$
$$Q(u,A,B) = u - 25A^2 - 25B^2$$

Properties:

- $x(u,A,B) + y(u,A,B) - z(u,A,B) - w(u,A,B) = 0$
- $y(A,A,1) + 7z(A,A,1) \equiv 0 \pmod{11}$
- $3[P(u,A,B) - Q(u,A,B) - 2t_{4,A}]$ is a nasty number
- $\{5[z(u,A,(A+1)(2A+1)) - w(u,A,(A+1)(2A+1))] + P(u,A,(A+1)(2A+1)) - Q(u,A,(A+1)(2A+1)) - 168p_A^4\}$ is a perfect square
- $z(A(A+1)(A+2),A,B) + w(A(A+1)(A+2),A,B) = P((A(A+1)(A+2),A,B) + Q(A(A+1)(A+2),A,B) = 12p_A^3$

Pattern –III

(3) can be written as $2r^2 - s^2 = v^2 * 1$ (10)

Write $v = 2a^2 - b^2$, $1 = (\sqrt{2}+1)(\sqrt{2}-1)$ (11)

Using (11) in (10) and employing the method of factorization, define

$$(\sqrt{2}r + s) = (\sqrt{2}a + b)^2(\sqrt{2}+1)$$

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Equating the rational and irrational parts, we have

$$s = 2a^2 + b^2 + 4ab$$

$$r = 2a^2 + b^2 + 2ab$$

Substitution in (2), the corresponding integer solutions to (1) are given by

$$x = u + 2a^2 - b^2$$

$$y = u - 2a^2 + b^2$$

$$z = u + 2a^2 + b^2 + 4ab$$

$$w = u - 2a^2 - b^2 - 4ab$$

$$P = u + 2a^2 + b^2 + 2ab$$

$$Q = u - 2a^2 - b^2 - 2ab$$

Properties:

- $[w(u,a,b) + P(u,a,b)]^2 - [x(u,a,b) + y(u,a,b)]^2 - [Q(u,a,b) - w(u,a,b)]^2 \equiv 0 \pmod{8u}$
- $x(u(u+1), A, B) + y(u(u+1), A, B) = 4t_{3,u}$
- $z(u, A(A+1), (A+2)(A+3)) - P(u, A(A+1), (A+2)(A+3)) = 48pt_A$
- $3[P(A(2A^2+1), A, B) + Q(A(2A^2+1), A, B)] = 2OH_A$
- $3[z(u,A,A) - P(u,A,A)] = 3[Q(u,A,A) - w(u,A,A)]$ is a nasty number

Pattern –IV

Write 1 as $1 = \frac{(5\sqrt{2}+1)(5\sqrt{2}-1)}{49}$

Repeating the above process as in pattern (3), the non-zero distinct integral solutions of (1) are found to be

$$x(u, A, B) = u + 98A^2 - 49B^2$$

$$y(u, A, B) = u - 98A^2 + 49B^2$$

$$z(u, A, B) = u + 14A^2 + 7B^2 + 140AB$$

$$w(u, A, B) = u - 14A^2 - 7B^2 - 140AB$$

$$P(u, A, B) = u + 70A^2 + 35B^2 + 14AB$$

$$Q(u, A, B) = u - 70A^2 - 35B^2 - 14AB$$

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Properties:

- $x(u^2(u+1),A,B)+y(u^2(u+1),A,B)=P(u^2(u+1),A,B)+Q(u^2(u+1),A,B)=4p_u^5$
- $P(u(2u^2-1),A,B)+Q(u(2u^2-1),A,B)=2so_u$
- $x(u(u+1),A,B)+y(u(u+1),A,B)=2pr_u$
- $3(z(u^2,A,B)+w(u^2,A,B))$ is a nasty number

CONCLUSION:

To conclude, one may search for other patterns of solutions and their corresponding properties.

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