THE EXPERIMENT
INTERNATIONAL JOURNAL OF SCIENCE AND TECHNOLOGY

ISSN-2319-2119

RESEARCH ARTICLE

Zakaria Hossain, The *Experiment, Jan. 2013 , Vol. .6(3), 346-354*

GAMMA MODULES WITH GAMMA RINGS OF GAMMA ENDOMORPHISMS

ABSTRACT

In this paper, we study gamma rings of gamma endomorphism by setting gamma modules. Some properties of these gamma rings of gamma endomorphisms are developed here. At last we have also obtained some significant results of prime Γ -rings and primitive Γ -rings.

KEY WORDS Gamma Endomorphisms, Gamma Modules, Irreducible Gamma rings, Gamma homorphisms, Gamma Isomorphism.

2010 AMS Subject Classification : Primary 16 N 60, Secondary 16 W 25, 16 U 80.

1. INTRODUCTION

The concepts of a Γ -ring was first introduced by Nobusawa [5] in 1964. Now a day his Γ -ring is called a Γ -ring in the sense of Nabusawa. This Γ -ring is generalized by W. E. Barnes [1] in a broad sense that served now –a-day to call a Γ -ring. A.C Paul and Md. Sabur Uddin [7] have developed some basic properties of modules in

Γ -ring M-modules. They have studied irreducible Γ M-module, the schur's Lemma and the Jordan-Holder Theorem in Γ M-modules. Three Isomorphism Theorems in Γ M-modules are studied by them. W.E. Barnes introduced the notation of Γ -homorphisms. prime and primary ideals, m-systems and the radical of Γ -rings. A.C. Paul and Md. Sabur Uddin [6] have also studied free Γ -modules over Γ -rings or division Γ -rings. They have developed in variant rank properties and cardinalities of these modules. A.C. Paul and Md. Sabur Uddin [8] generalized free Γ -modules over Γ -principal ideal domains They have also developed some properties of these Γ -modules.

In classical ring theories, N.H. McCoy [4] studied rings of Endomorphisms by setting M-modules.

In this paper, we have generalized the results of N.H.McCoy [4] into Γ -rings of Γ -endomorphisms by using Γ -modules.

2. Preliminaries

2.1 Gamma Ring

Let M and Γ be two additive abelian groups. Suppose that there is a mapping from $M \times \Gamma \times M \to M$ (sending (x, α, y) into $x\alpha y$) such that

(i) $(x + y)\alpha z = x\alpha z + y\alpha z$

$x(\alpha + \beta)z = x\alpha z + x\beta z$

$x\alpha(y + z) = x\alpha y + x\alpha z$

(ii) $(x\alpha y)\beta z = x\alpha(y\beta z)$,

where $x, y, z \in M$ and $\alpha, \beta \in \Gamma$. Then M is called a Γ-ring in the sense of Barnes [1].

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2.2 Unity element of a Γ-ring

Let M be a Γ-ring. M is called a Γ-ring with unity if there exists an element $e \in M$ such that $a\gamma e = e\gamma a = a$ for all $a \in M$ and some $\gamma \in \Gamma$. We shall frequently denote e by 1 and when M is a Γ-ring with unity, we shall often write $1 \in M$. Note that not all Γ-rings have an unity. When a Γ-ring has an unity, then the unity is unique.

2.3 Ideal of Γ-rings

A subset A of the Γ-ring M is a left (right) ideal of M if A is an additive subgroup of M and $M\Gamma A = \{c\alpha a \mid c \in M, \alpha \in \Gamma, a \in A\}(A\Gamma M)$ is contained in A. If A is both a left and a right ideal of M, then we say that A is an ideal or two-sided ideal of M.

It is clear that the intersection of any number of left (respectively right or two-sided) ideal of M is also a left (respectively right or two-sided) ideal of M.

If A is a left ideal of M, B is a right ideal of M and S is any non-empty subset of M, then the set, $A\Gamma S = \{\sum_{i=1}^{n} a_i\gamma s_i \mid a_i \in A, \gamma \in \Gamma, s_i \in S$, n is a positive integer$\}$ is a left ideal of M and SΓB is a right ideal of M. AΓB is a two-sided ideal of M.

If $a \in M$, then the principal ideal generated by a denoted by $\langle a \rangle$ is the intersection of all ideals containing **a** and is the set of all finite sum of elements of the form $na + x\alpha a + a\beta y + u\gamma a\mu v$, where n is an integer, x, y, u, v are elements of M and α, β, γ, μ are elements of Γ. This is the smallest ideal generated by a. Let $a \in M$. The smallest left (right) ideal generated by a is called the principal left (right) ideal $\langle a|$ $(| a\rangle)$.

2.4 Γ -homomorphism

Let M and S be two Γ-rings. A mapping θ of a Γ-ring M into

a Γ-ring S is said to be a Γ-homomorphism of M into S if addition and multiplication are preserved under this mapping, that is, if $a, b \in M$, $\alpha \in \Gamma$

(i) $(a+b)\theta = a\theta + b\theta$

(ii) $(a\alpha b)\theta = (a\theta)\alpha(b\theta)$.

If θ is one-one and onto then θ is called a Γ-isomorphism from M into S.

2.5 Kernel of Γ-homomorphism

If θ is a homomorphism of a Γ-ring M into a Γ-ring S, then $(0)\theta^{-1}$, that is, the set of all elements a of M such that $a\theta = 0$ (the zero of S), is called the kernel of the Γ-homomorphism θ.

2.6 Prime ideal

An ideal K of a Γ-ring M is prime if $K \neq M$ and for any ideals

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A and B, AΓB $\subseteq$ K , then A $\subseteq$ K or B $\subseteq$ K.

2.7 Prime Γ-ring

A Γ-ring M is said to be a prime Γ-ring if and only if the zero ideal is a prime ideal in M.

2.8 Fundamental Theorem on Γ-homorphisms

Let θ be a Γ-homomorphism of the Γ-ring M onto the Γ-ring $S = \mathrm{M}\theta$, with kernel K. Then K is an ideal in M and $S \cong \mathrm{M}/\mathrm{K}$. In fact, the mapping $\varphi : (a\theta)\varphi = a + \mathrm{K}, a \in \mathrm{M}$ is a Γ-isomorphic of S onto M/K .

2.9 Minimal right (left) ideal of a Γ-ring

Let M be a Γ-ring. A right (left) ideal A of M is called a minimal right (left) ideal if

(i) A $\neq$ 0

(ii) Whenever A $\supseteq$ B $\supseteq$ 0, B is a right (left) ideal of M, then either B = A or B = 0.

It is clear that if a Γ-ring M $\neq$ 0 satisfies the minimum condition on right (left) ideals, then M has a minimal right (left) ideal.

2.10 Maximal ideal

An ideal R in a Γ-ring M is called a maximal ideal in M if (i)R$\subset$M and (ii) whenever L is an ideal in M such that R $\subseteq$ L $\subseteq$ M, then either L = R or L = M.

3. Gamma Modules with Gamma Rings of Gamma Endomorphisms

3.1 Definition

A mapping a: $x \rightarrow x\alpha a,\ x \in V, \alpha \in \Gamma$, of the abelian group V into itself is called a Γ -endomorphism of V if for $x, y \in V$, we have $(x+y)\alpha a = x\alpha a + y\alpha a$.....(1)

3.2 Definition

Let N be an additively written abelian group and M be a Γ -ring. Then N is said to be a (right) Γ M-module if a law of composition of N× Γ ×M into N is defined (that is, if $x \in \mathrm{N}, \alpha \in \Gamma, a \in \mathrm{M}, x\alpha a$ is a uniquely determine element of N) such that the following are true for $x, y \in \mathrm{N}, \alpha, \beta \in \Gamma$ and $a, b \in \mathrm{M}$:

$$(i)\ (x+y)\alpha a = x\alpha a + y\alpha a$$

$$(ii)\ x\alpha(a+b) = x\alpha a + x\alpha b$$

$$(iii)\ x\alpha(a\beta b) = (x\alpha a)\beta b.$$

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If M is given a Γ-ring of Γ-endomorphism of the abelian group N property (i) would hold by definiton of a Γ-endomorphism and the other two properties would hold by the definiton of addition and multiplication of Γ-endomorphisms. Actually, if N is a Γ M-module and a $\in \mathrm{M}$, property (i) assures us that the mapping $x \to x\alpha a, x \in \mathrm{N}, \alpha \in \Gamma$ is a uniquely determined

Γ-endomorphism of N associated with the element a of M. However, we shall see below that M need not be (Γ-isomorphic to) a Γ-ring of Γ-endomorphisms of N simply because different elements of M may be associated with the same Γ-endomorphism of N.

Let us point out that if N is a Γ M-module, each of the following is true for each $x \in \mathrm{N}, \alpha \in \Gamma$ and $a \in \mathrm{M}$:

$x\alpha 0 = 0, 0\alpha a = 0, (-x)\alpha a = x\alpha(-a) = -(x\alpha a)$.

If N is a Γ M-module and $a \in \mathrm{M}$, then the mapping $x \to x\alpha a, \alpha \in \Gamma$ of N into N is a

Γ-endomorphism of the abelian group N. Let us now denote this Γ-endomorphism of N by a^*, that is, the Γ-endomorphism a^* is defined by $x\alpha a^* = x\alpha a, x \in \mathrm{N}, \alpha \in \Gamma$.........(2).

Now $x\alpha(a+b)^* = x\alpha(a+b) = x\alpha a + x\alpha b = x\alpha a^* + x\alpha b^* = x\alpha(a^* + b^*)$, $x \in \mathrm{N}, \alpha \in \Gamma$.

Thus $(a+b)^* = a^* + b^*$. Again $x\alpha(a\beta b)^* = x\alpha(a\beta b) = (x\alpha a)\beta b = x\alpha a^* \beta b = (x\alpha a^*)\beta b$

$= x\alpha a^* \beta b^* = x\alpha(a^* \beta b^*)$. Hence $(a\beta b)^* = a^* \beta b^*$. Thus $\left.\begin{matrix}(a+b)^* = a^* + b^* \\ (a\beta b)^* = a^* \beta b^*\end{matrix}\right\}$..........(3).

It follows easily that the set $\mathrm{M}^* = \{a^* | a \in \mathrm{M}\}$.........(4) is actually a Γ-ring of Γ-endomorphisms of the abelian group N and that the mapping $a \to a^*, a \in \mathrm{M}$, is a Γ-homomorphism of M onto M^*. It is customary to denote the kernel of this Γ-homomorphism by (0:N). Thus we have (0:N)$= \{a | a \in \mathrm{M}, x\alpha a = 0 \textit{ for every } x \in \mathrm{N}, \alpha \in \Gamma\}$.......(5). Clearly, by this definition (0:N) is an ideal of M and $x\alpha c = 0$ for every $x \in \mathrm{N}, \alpha \in \Gamma$ and $c \in (0:N)$. The Fundamental theorem on Γ-homomorphisms (2.8) now yeilds immediately the following result.

3.3 Theorem

If N is a Γ M-module, then the Γ-ring $\mathrm{M}/(0:\mathrm{N})$ is Γ-isomorphic to a Γ-ring M^* of Γ-endomorphisms of the abelian group N.

Thus, if N is a Γ M-module, some Γ-homomorphic image of M (M^* is the notation above) is Γ-ring of Γ-endomorphisms of the abelian group N. In the view of (2) and known properties of Γ-endomorphisms, we see that N may be also be considered to be a Γ M^*-module if we with to do so.

We shall now discuss in turn two special cases that are particularly important for our purposes.

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First, let us start with a Γ-ring M and let A be a right ideal in M. For the moment, let us denote by N the additive group A^+ of the ring A. For $x \in N$ (which means the same as $x \in A^+$ or $x \in A$ for the matter) and $a \in M$, we consider $x\alpha a, \alpha \in \Gamma$ to be the product in M of the elements x and a of M. In this case the three properties of Definition 3.2 are Γ-ring properties, so that N is clearly a Γ M-module. It will now be convenient to denote (0:N) by (0:A) and hence $(0:A) = \{m | m \in M, A\Gamma m = 0\}$.

3.4 Definition

Let M be a nonzero Γ-ring of Γ-endomorphisms of the abelian group V. If the only Γ M-subgroup W of V are $W = 0$ and W = V, we say that M is an irreducible

Γ-ring of Γ-endomorphisms of V.

3.5 Lemma

Let M be a non zero Γ-ring of Γ-endomorphisms of an abelian group V, then M is an irreducible Γ-ring of Γ-endomorphisms of V if and only if $x\Gamma M = V$ for every nonzero element x of V.

Proof

One part is essentially trivial. For if $x\Gamma M = V$ for every nonzero element x of V, it is clear that V is the only nonzero Γ M-subgroup of V and M is therefore irreducible.

Conversely, let us assume that M is an irreducible Γ-ring of Γ-endomorphisms of V and that x is an arbitrary nonzero element of V. It is easily verified that $x \Gamma M$ is a subgroup of V and since $(x\Gamma M)\Gamma M \subseteq x\Gamma M$, it is a Γ M-subgroup of V.

It is follows that $x\Gamma M = 0$ or $x\Gamma M = V$ suppose that $x\Gamma M = 0$ and let $< x >$ be the subgroup of V generated by x (that is, $< x > = \{nx | n \text{ is an integer}\}$). Then $< x > \Gamma M = 0$ and therefore $< x >$ is a Γ M-subgroup of V. Since $x \in < x >$ and $x \neq 0$ we must have $< x > = V$ and therefore $V\Gamma M = 0$. However, this is impossible since M has nonzero elements and the assumption that $x\Gamma M = 0$ has led to a contradiction. Hence $x\Gamma M = V$ and the proof is completed.

We shall now prove the following result.

3.6 Theorem

If A is a minimal right ideal in the Γ-ring M such that $A\Gamma A \neq 0$, then the

Γ-ring $M/(0:A)$ Γ-isomorphic to an irreducible Γ-ring of Γ-endomorphisms of the abelian group A^+.

Proof

If in the present setting we let M^* be the Γ-ring defined as in (4), the preceding theorem shows that we only need to prove that the Γ-ring M^* of Γ-endomorphisms of the abelian group A^+ is irreducible.

THE EXPERIMENT
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ISSN-2319-2119

RESEARCH ARTICLE

Zakaria Hossain, The *Experiment, Jan. 2013 , Vol. .6(3), 346-354*

Since $A\Gamma A \neq 0$ is clear from (2) that M^* must contain nonzero elements. Let a be an arbitrary nonzero element of A^+. We make use of Lemma 3.5 and complete the proof by showing that $a\Gamma M^* = A^+$. In view of (2), this is equivalent to showing that $a\Gamma M = A$. Since $a \in A$, $a\Gamma M$ is a right ideal of M which is contained in the minimal right ideal A. Accordingly, we must have either $a\Gamma M = 0$ or $a\Gamma M = A$. Suppose that $a\Gamma M = 0$. Then the set $\{c | c \in A, c\Gamma M = 0\}$ is a right ideal of M contained in A; it contains the nonzero element a and hence it must coincide with A, that is $A\Gamma M = 0$. However, this contradicts the assumption that $A\Gamma A \neq 0$. Accordingly, we conclude that $a\Gamma M = A$, and the proof is completed. It is easy to apply this result to establish the next theorem 3.8.

3.7 Primitive Γ-ring

A Γ-ring which is Γ-isomorphic to an irreducible Γ-ring of

Γ-endomorphisms of some abelian group may be called a primitive Γ-ring.

3.8 Theorem

A prime Γ-ring which contains a minimal right ideal is a primitive Γ-ring.

Proof

Let A be a minimal right ideal in the prime Γ-ring. Now $A \neq 0$ by definition of minimal right ideal. It follows that $A\Gamma A \neq 0$ since M is a prime Γ-ring. Moreover, by definition of the ideal (0:A) in M, we have that $A\Gamma(0{:}A) = 0$. Since $A \neq 0$, we conclude that $(0{:}A) = 0$, and therefore $M \cong M/(0:A)$. The preceding theorem then shows that M is a primitive Γ-ring and this completes the proof.

We now consider another important special class of ΓM-modules.

Let N is the additive group whose elements are the cosets $x + A, x \in M$, with addition defined by $(x + A) + (y + A) = (x + y) + A, \quad x, y \in M$.........(6). The zero of N is then the zero coset $0 + A = A$. We now defined a composition of $N \times \Gamma \times M$ into N by $(x + A)\gamma m = x\gamma m + A, \quad x + A \in N, \gamma \in \Gamma, m \in M$.............(7) .

It is easy verify that under the definition N is a ΓM-module. Let us see what the ideal (0: N) is in this case. By definition of (0:N), we have $(0{:}N) = \{m | m \in M, (x + A)\alpha m = 0$ for every $x + A \in M, \alpha \in \Gamma\}$. In view of Definition (7) and the fact that the zero of N is the coset A, this becomes $(0{:}\ N) = \{m | m \in M, M\Gamma m \subseteq A\}$.

It is customary to denote this ideal by (A:M), that is , to repeat the definition, $(A{:}M) = \{m | m \in M, M\Gamma m \subseteq A\}$.

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Zakaria Hossain, The *Experiment, Jan. 2013 , Vol. .6(3), 346-354*

In this particular setting the Γ-ring M^* of Γ-endomorphisms of the abelian group $N = M^+ - A^+$consests of the Γ-endomorphisms m^*, $m \in M$, defined by $(x + A)\alpha m^* = x\alpha m + A,\ x + A \in N,\ \alpha \in \Gamma$.........(8) , and theorem 3.3 assures us that $M^* \cong M/(A:M)$.

An important special case is that in which the right ideal A is a maximal right ideal. First, let us prove the following result.

3.9 Theorem

If A is a maximal right ideal in the Γ-ring M and such that $M\Gamma M \not\subset A$ then the Γ-ring $M/(A:M)$ is Γ-isomorphic to an irreducible Γ-ring of Γ-endomorphisms of abelian group $M^+ - A^+$.

Proof

The fact that $M\Gamma M \not\subset A$ assures us that $(A:M) \neq M$ and therefore that the Γ-ring $M/(A:M)$ has nonzero elements. In the notation used above, M^* is therefore a nonzero

Γ-ring of Γ-endomomorphisms of the abelian group $N = M^+ - A^+$. We proceed to show that it is irreducible. Clearly B $= \{t \mid t \in M,\ t\Gamma M \subseteq A\}$ is a right ideal in M such that $A \subseteq B$. Moreover, $B \neq M$ since $M\Gamma M \not\subset A$, and the maximality of A implies that $B = A$. Now let s be an arbitrary element of M which is not an element of A. Since $s \notin B$, $s\Gamma B \not\subset A$ and again using the maximality of A, we have $s\Gamma M + A = M$. It follows from (8) that $(s + A)\Gamma M^* = N$. The condition $s \notin A$ is equivalent to the condition that $s + A$ is not the zero element of N. Accordingly, Lemma 3.5 shows that M^* is an irreducible Γ-ring of Γ-endomorphisms of the abelian group $N = M^+ - A^+$, and the proof of the theorem is completed.

We next the following definition.

3.10 Definition

A right ideal A in a Γ-ring M is said to be modular right ideal if there exists an element e of M such that $e\alpha m - m \in A$ for every $m \in M,\ \alpha \in \Gamma$.

It is clear that if M has an unity (or just a left for that matter), then every right ideal in M is modular. A maximal right ideal which is also a modular right ideal will naturally be called a modular maximal right ideal. We shall now prove the following theorem.

3.11 Theorem

If A is a modular maximal right ideal in the Γ-ring M, then $M/(A:M)$ is a primitive Γ-ring. The Γ-ring M is itself a primitive Γ-ring if and only if it contains a modular maximal right ideal A such that $(A:M) = 0$.

ISSN-2319-2119

RESEARCH ARTICLE

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Proof

Suppose, first that A is a modular maximal right ideal in M. There exists an element e of M such that $e\alpha m - m \in A$ for all m in M $\alpha \in \Gamma$ Now $A \neq M$ since A is a maximal right ideal in M, hence there exists $s \in M$ such that $s \notin A$. Then $e\alpha s \notin A$ and this shows that $M\Gamma M \not\subset A$. The first statement of the theorem the follows immediately from theorem 3.9.

If $(A;M) = 0$, then $M \cong M/(A{:}M)$ and one part of the second statement is a consequence of what we have just proved.

To prove the other part, suppose that M is an irreducible Γ-ring of Γ-endomorphisms of an abelian group V. Let x be a fixed nonzero element of V, and let us set $A = \{a | a \in M,\ x\alpha a = 0, \alpha \in \Gamma\}$. We shall complete the proof by showing that A is a modular maximal right ideal of M and that $(A{:}M) = 0$. Clearly, A is a right ideal in M. Since $x \neq 0$. Lemma 3.5 shows that $x\Gamma M = V$ and therefore $A \neq M$. To show that A is maximal suppose that $c \in M, c \notin A$ and let $B = A + < c >$. Since $x\alpha c \neq 0,\ \alpha \in \Gamma, x\Gamma B \neq 0$. But $(x\Gamma B)\Gamma M \subset x\Gamma B$ and the irreducibility of M shows that we must have $x\Gamma B = V$. Therefore for each element s of M there must exist $b \in B$ such that $x\alpha b = x\alpha s, \alpha \in \Gamma$. It follows that $x\alpha(b-s) = 0$ and $b - s \in A$. Since $A \subseteq B$, we see that $s \in B$ and therefore that $M = B = A + < c >$. Since this is true for every element c of M which is not in A, A is indeed a maximal right ideal in M. Now $x\Gamma M = V$ implies that there exists an element e of M such that $x\alpha e = x$, $\alpha \in \Gamma$. Then $x\alpha e\alpha m = x\alpha m,\ \alpha \in \Gamma$ for every element m of M, that is, $x\alpha(e\alpha m - m) = 0$ and $e\alpha m - m \in A$. This shows that A is modular and there remains only to verify that $(A{:}M) = 0$. If $a \in (A : M)$, then $M\Gamma a \subseteq A$ and hence $x\Gamma M\Gamma a = 0$. Since $x\Gamma M = V$, it follows that $V\Gamma a = 0$ and this implies that $a = 0$. Hence $(A{:}M) = 0$ and the proof is completed.

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